Appendix A - Model equations

Firms Transactions and Profit

$Y_d = C + IN + G - C_{INT}$ (1) Aggregate demand (constant prices, 2010 USD)

$Y_s = Y_d$ (2) Aggregate supply (equilibrium condition)

$NY = Y_s \cdot \frac{p}{p_{[2010]}}$ (3) Nominal output (current prices)

$C_{INT} = c_0 + c_1 \cdot C_{INT,-1}$ (4) Intermediate consumption

$WB = w \cdot N \cdot H$ (5) Wage bill

$\omega = \frac{WB}{Y}$ (6) Wage share

$L_f = L_{f,-1} + I_f + BE - C_{INT} - d(e_s) \cdot p_e - F_{fu}$ (7) Loans to firms

$F_f = Y_d - WB - r_{l,-1} \cdot L_{f,-1} - TAX_f$ (8) Firms' total profit (net of taxes)

$F_{fu} = F_f \cdot \theta$ (9) Firms' retained profit

$F_{fd} = F_f - F_{fu}$ (10) Firms' distributed profit

Firms Investment Decisions

$K_c = K_{c,-1} + I_c - DA_c$ (11) Conventional capital stock

$I_f = h \cdot E(Y_d) \cdot \left(1 - d_{T,-1}\right)$ (12) Total private investment

$u = u_{-1} \cdot \frac{1+g_Y}{(1+g_K)\cdot(1+g_{Af})}$ (13) Actual utilisation rate of plants (note: $0 < u \leq 1$ and $g_{Af} = \Delta a_f / a_f$)

$h = [1 + \phi \cdot (u_{-1} - u_n)] \cdot h_{-1}$ (14) Total investment share to output

$I_c = I_f - I_{gr}$ (15) Conventional investment undertaken by firms

$DA_c = \delta_c \cdot K_{c,-1}$ (16) Depreciation allowances on conventional capital

$I_{gr} = \gamma_{gr} \cdot I_f$ (17) Green private investment

$\gamma_{gr} = \gamma_0^{gr} + \gamma_1^{gr} \cdot \gamma_{gr,-1} + \gamma_2^{gr} \cdot \frac{G_{gr,-1}}{G_{-1}} + \gamma_3^{gr} \cdot d_{T,-1} + \gamma_4^{gr} \cdot \frac{C_{gr,-1}}{C_{-1}} + \gamma_5^{gr} \cdot \tau_{f,-1}$ (18) Share of green investment to total investment

$K_{gr} = K_{gr,-1} + I_{gr} - DA_{gr}$ (19) Green capital stock

$K_f = K_c + K_{gr}$ (20) Total stock of capital at constant prices

$DA_{gr} = \delta_{gr} \cdot K_{gr,-1}$ (21) Depreciation allowances on green capital

$$DA_f = DA_c + DA_{gr}$$ (22) Total depreciation allowances

$$BE = \gamma_0^{tech} + \gamma_1^{tech} \cdot G_{tech,-1}$$ (23) Private non-green innovative spending

$$IN = I_f + BE$$ (24) Total spending for investment and innovation

$$e_s = e_{s,-1} + \chi \cdot \frac{I_{f,-1}}{p_{e,-1}}$$ (25) Quantity of new shares issued by firms as a percentage of planned investment

$$g_Y = \frac{\Delta Y}{Y_{-1}}$$ (26) Real output growth rate

$$g_{NY} = \frac{\Delta NY}{NY_{-1}}$$ (27) Nominal output growth rate

$$g_K = \frac{\Delta K_f}{K_{f,-1}}$$ (28) Rate of accumulation of total capital

Households Income and Wealth

$$YD_w = WB + r_{d,-1} \cdot D_{w,-1} - T_w - r_{l,-1} \cdot L_{w,-1} - VAT_w$$ (29) LC disposable income

$$YD_\pi = F_{fd} + F_b + r_{d,-1} \cdot D_\pi + r_{b,-1} \cdot B_{d,-1} - T_\pi - VAT_\pi$$ (30) UC's disposable income

$$YD_\pi^{hs} = YD_\pi + CG$$ (31) UC's Haig-Simons disposable income

$$YD = YD_w + YD_\pi$$ (32) Total disposable income

$$S_w = YD_w - C_w$$ (33) LC's saving

$$NW_w = NW_{w,-1} + S_w$$ (34) Net wealth of LC

$$L_w = L_{w,-1} \cdot (1 - rep) + \psi \cdot C_w$$ (35) Loans to LC

$$bur = \frac{L_{w,-1} \cdot (rep + r_{l,-1})}{YD_w}$$ (36) Debt burden of LC

$$NW_\pi = NW_{\pi,-1} + YD_\pi^{hs} - C_\pi$$ (37) Net wealth of UC

$$NW = NW_w + NW_\pi$$ (39) Total net wealth of households

Households Consumption Decisions

$$C_w = \left[c_w \cdot YD_w \cdot \frac{E(p_w)}{p_w} + c_{aw} \cdot NW_{w,-1} \cdot \frac{E(p_w)}{p_w}\right] \cdot \left(1 - d_{T,-1}\right)$$ (40) Total consumption of LC (net of climate-related damages)

$$C_\pi = \left[c_\pi \cdot YD_\pi^{hs} \cdot \frac{E(p_\pi)}{p_\pi} + c_{a\pi} \cdot NW_{\pi,-1} \cdot \frac{E(p_\pi)}{p_\pi}\right] \cdot \left(1 - d_{T,-1}\right)$$ (41) Total consumption of UC (net of climate-related damages)

$$C_{gr}^w = c_{gr}^w \cdot C_w$$ (42) Green consumption of LC

$C_{gr}^{\pi} = c_{gr}^{\pi} \cdot C_{w}^{\pi}$ (43) Green consumption of UC

$C_{c}^{w} = C_{w} - C_{gr}^{w}$ (44) Routine consumption of LC

$C_{c}^{\pi} = C_{\pi} - C_{c}^{w}$ (45) Routine consumption of UC

$c_{gr}^{w} = c_{0}^{w} + c_{1}^{w} \cdot d_{T,-1} + c_{2}^{w} \cdot \left(vat_{c} - vat_{gr}\right)$ (46) Green consumption share of LC

$c_{gr}^{\pi} = c_{0}^{\pi} + c_{1}^{\pi} \cdot d_{T,-1} + c_{2}^{\pi} \cdot \left(vat_{c} - vat_{gr}\right)$ (47) Green consumption share of UC

$C = C_{w} + C_{\pi}$ (48) Total consumption

$C_{gr} = C_{gr}^{w} + C_{gr}^{\pi}$ (49) Total green consumption

Households Portfolio Decisions

$p_{e} = E(NW_{\pi}) \cdot \left[\lambda_{10} + \lambda_{11} \cdot E(r_{e}) + \lambda_{12} \cdot \frac{E(YD_{\pi})}{E(NW_{\pi})} + \lambda_{13} \cdot E(r_{b}) + \lambda_{14} \cdot E(r_{d})\right] \cdot \frac{1}{e_{d}}$ (50) Unit price of shares

$e_{d} = e_{s}$ (51) Equilibrium condition for the stock market

$E_{d} = e_{d} \cdot p_{e}$ (52) Nominal shares held by capitalist households

$B_{d} = E(NW_{\pi}) \cdot \left[\lambda_{20} \cdot + \lambda_{21} \cdot E(r_{e}) + \lambda_{22} \cdot \frac{E(YD_{\pi})}{E(NW_{\pi})} + \lambda_{23} \cdot E(r_{b}) + \lambda_{24} \cdot E(r_{d})\right]$ (53) Nominal government bills held by capitalist households

$D_{\pi} = E(NW_{\pi}) \cdot \left[\lambda_{30} + \lambda_{31} \cdot E(r_{e}) + \lambda_{32} \cdot \frac{E(YD_{\pi})}{E(NW_{\pi})} + \lambda_{33} \cdot E(r_{b}) + \lambda_{34} \cdot E(r_{d})\right]$ (54) Deposits held by capitalist households

$H_{\pi} = NW_{\pi} - E_{d} - B_{d} - D_{\pi}$ (55) Cash held by capitalist households

$D_{w} = NW_{w}^{G} - H_{w}$ (56) Deposits held by LC

$NW_{w}^{G} = NW_{w} + L_{w}$ (57) Gross wealth of LC

$H_{w} = \lambda_{w} \cdot NW_{w}^{G}$ (58) Cash held by LC

$D_{d} = D_{w} + D_{\pi}$ (59) Total demand for bank deposits

$H_{d} = H_{w} + H_{\pi}$ (60) Total demand for cash

Commercial Banks and Central Bank

$D_{s} = D_{d}$ (61) Supply of bank deposits

$A_{d} = D_{s} - L_{s} + H_{d}^{B}$ (62) Demand for advances (+) / Excess reserves (–)

$A_{s} = A_{d}$ (63) Supply of advances (+) / Excess reserves (–)

$H_{d}^{B} = \rho_{B} \cdot D_{s,-1}$ (64) Reserve requirement (demand)

$H_s^B = H_d^B$ (65) Reserve requirement (supply)

$L_s = L_{s,-1} + d(L_d)$ (66) Supply of loans (endogenous)

$L_d = L_f + L_w$ (67) Total demand for loans

$F_b = L_{s,-1} \cdot r_{l,-1} - D_{s,-1} \cdot r_{d,-1}$ (68) Bank profit

$B_{cb} = B_s - B_d$ (69) T-bills purchased by CB (residual amount)

$H_s = B_{cb} + A_s - H_s^B$ (70) Money created by CB

$r_l = r_{cb} + \mu_l$ (71) Interest rate on bank loans

$r_d = r_{cb}$ (72) Return rate on bank deposits

Other Financial Variables and Indices

$CG = e_{s,-1} \cdot d(p_e)$ (73) Capital gains/losses on shares

$r_e = \frac{F_f}{e_{s,-1} \cdot p_{e,-1}}$ (74) Dividend yields

$q = \frac{e_s \cdot p_e + L_f}{K_f}$ (75) Tobin's q

$\ell = \frac{L_f}{e_s \cdot p_e + L_f}$ (76) Firms' leverage ratio

$per = \frac{p_e}{F_f / e_{s,-1}}$ (77) Price-earnings ratio

Government Spending and Taxation

$TAX = TAX_f + TAX_w + TAX_\pi + VAT_w + VAT_\pi$ (78) Total tax revenue

$VAT_w = C_w \cdot \frac{vat_w}{1 + vat_w}$ (79) Taxes on value added paid by LC

$VAT_\pi = C_\pi \cdot \frac{vat_\pi}{1 + vat_\pi}$ (80) Taxes on value added paid by UC

$vat_w = vat_{gr} \cdot \frac{C_{gr}^w}{C_w} + vat_c \cdot \frac{C_c^w}{C_w}$ (81) Average VAT rate for LC

$vat_\pi = vat_{gr} \cdot \frac{C_{gr}^\pi}{C_\pi} + vat_c \cdot \frac{C_c^\pi}{C_\pi}$ (82) Average VAT rate for UC

$TAX_f = \tau_f \cdot emis_{in,-1}$ (83) Taxes on firms' emissions (carbon tax)

$TAX_w = \tau_w \cdot \left(WB + r_{d,-1} \cdot D_{w,-1}\right)$ (84) Taxes on LC's income

$TAX_\pi = \tau_\pi \cdot \left(F_{fd} + F_b + r_{d,-1} \cdot D_{\pi,-1} + r_{b,-1} \cdot B_{d,-1}\right)$ (85) Taxes on UC's income (excluding capital gains)

$G = G_{rout} + G_{mois}$ (86) Total government spending (net of interest payments)

$G_{rout} = G_{rout,-1} \cdot (1 + g_G^{rout})$ (87) Routine government spending

$G_{mois} = G_{mois,-1} \cdot \left(1 + g_G^{mois}\right)$ (88) Mission-oriented innovation spending by government (MOIS)

$G_{gr} = \alpha \cdot G_{mois}$ (89) Government MOIS devoted to green conversion

$G_{tech} = (1 - \alpha) \cdot G_{mois}$ (90) Other government MOIS (e.g., new technologies)

Government Budget

$B_s = B_{s,-1} + GDEF$ (91) Nominal supply of government bills

$GDEF = G + r_{b,-1} \cdot (B_{s,-1} - B_{cb,-1}) - T$ (92) Government deficit (note: no interest payments on government bills held by CB)

$GDEB = GDEB_{-1} + GDEF$ (93) Stock of government debt

$r_b = r_{cb} + \mu_b$ (94) Return rate on government bills

$\mu_b = \eta_0 + \eta_1 \cdot d_{T,-1}$ (95) Risk premium on T-bills

The Ecosystem: Material Resources and Reserves

$y_{mat} = \mu \cdot Y_s$ (96) Production of material goods

$mat = y_{mat} - rec$ (97) Extracted matter

$rec = \rho_{rec} \cdot des$ (98) Recycled socio-economic stock

$des = \mu \cdot \left(DA_f + \zeta \cdot DC_{-1}\right)$ (99) Demolition or disposition of socio-economic stock

$DC = DC_{-1} \cdot (1 - \zeta) + C$ (100) Durable goods (lasting more than one period)

$k_{se} = k_{se,-1} + y_{mat} - des$ (101) Socio-economic stock

$wa = mat + cen + o2 - emis - \Delta k_{se} = mat - \Delta k_{se}$ (102) Waste generated by production process

$hws = hws_{-1} + haz \cdot wa$ (103) Hazardous waste level

$hratio = \frac{hws}{surf}$ (104) Hazardous waste ratio (Gt/Km^2)

$k_m = k_{m,-1} + conv_m - mat$ (105) Stock of material reserves

$conv_m = \max(\sigma_{m,-1} \cdot res_{m,-1}, mat_{-1})$ (106) Material resources converted to reserves

$res_m = res_{m,-1} - conv_m$ (107) Stock of material resources

$p_m = p_m^0 + p_m^1 \cdot \frac{mat_{-1}}{\sigma_{m,-1} \cdot res_{m,-1}}$ (108) Unit price of extracted matter

$\sigma_m = \sigma_m^0 + \sigma_m^1 \cdot E(p_m)$ (109) Actual conversion rate of matter resources

$cen = \frac{emis}{car}$ (110) Carbon mass of (non-renewable) energy

$o2 = emis - cen$ (111) Mass of oxygen (O_2)

The Ecosystem: Energy Resources and Reserves

$e = \varepsilon \cdot Y_s$ (112) Total energy required for production

$er = \eta_{en} \cdot e$ (113) Renewable energy at the end of the period

$en = e - er$ (114) Non-renewable energy

$ed = en + er$ (115) Dissipated energy at the end of the period

$k_{en} = k_{en,-1} + conv_{en} - en$ (116) Stock of energy reserves

$conv_{en} = \max(\sigma_{en,-1} \cdot res_{en,-1}, en_{-1})$ (117) Energy resources converted to reserves

$res_{en} = res_{en,-1} - conv_{en}$ (118) Stock of energy resources

$p_{en} = p_{en}^0 + p_{en}^1 \cdot \frac{en_{-1}}{\sigma_{en,-1} \cdot res_{en,-1}}$ (119) Unit price of energy

$\sigma_{en} = \sigma_{en}^0 + \sigma_{en}^1 \cdot E(p_{en})$ (120) Actual conversion rate of energy resources

Emissions and Climate Change

$emis_{in} = \beta \cdot en$ (121) Industrial emissions of CO_2

$emis_l = emis_{l,-1} \cdot (1 - g_l)$ (122) Land emissions of CO_2

$emis = emis_{in} + emis_l$ (123) Total emissions of CO_2

$co2_{AT} = emis + \psi_{11} \cdot co2_{AT,-1} + \psi_{21} \cdot co2_{UP,-1}$ (124) Atmospheric CO_2 concentration

$co2_{UP} = \psi_{12} \cdot co2_{AT,-1} + \psi_{22} \cdot co2_{UP,-1} + \psi_{32} \cdot co2_{LO,-1}$ (125) Upper ocean / biosphere CO_2 concentration

$co2_{LO} = \psi_{23} \cdot co2_{UP,-1} + \psi_{33} \cdot co2_{LO,-1}$ (126) Lower ocean CO_2 concentration

$F = F_2 \cdot \log_2\left(\frac{co2_{AT}}{co2_{AT}^{PRE}}\right) + F_{ex}$ (127) Radiative forcing over pre-industrial levels (W/m^2)

$F_{ex} = F_{ex,-1} + fex$ (128) Radiative forcing (W/m^2) due to non-CO_2 greenhouse gases

$$T_{AT} = T_{AT,-1} + t_1 \cdot \left[F - \frac{F_2}{sens} \cdot T_{AT,-1} - t_2 \cdot \left(T_{AT,-1} - T_{LO,-1}\right)\right]$$ (129) Atmospheric temperature (C)

$$T_{LO} = T_{LO,-1} + t_3 \cdot \left(T_{AT,-1} - T_{LO,-1}\right)$$ (130) Lower ocean temperature (C)

Ecological Efficiency

$$\mu = \mu_{gr} \cdot \frac{K_{gr}}{K_f} + \mu_c \cdot \frac{K_c}{K_f}$$ (131) Matter-intensity coefficient

$$\varepsilon = \varepsilon_{gr} \cdot \frac{K_{gr}}{K_f} + \varepsilon_c \cdot \frac{K_c}{K_f}$$ (132) Energy-intensity coefficient

$$\beta = \beta_{gr} \cdot \frac{K_{gr}}{K_f} + \beta_c \cdot \frac{K_c}{K_f}$$ (133) CO_2-intensity coefficient

$$\eta_{en} = \eta_{gr} \cdot \frac{K_{gr}}{K_f} + \eta_c \cdot \frac{K_c}{K_f}$$ (134) Share of renewable energy sources

$$\rho_m = \frac{mat}{k_{m,-1}}$$ (135) Matter depletion ratio (net of recycling)

$$\rho_{en} = \frac{en}{k_{en,-1}}$$ (136) Non-renewable energy depletion ratio

Ecological Feedbacks and Damages

$$\delta_c = \delta_c^0 + (1 - \delta_c^0) \cdot (1 - ad_K^c) \cdot d_{TF,-1}$$ (137) Impact of climate change on conventional capital stock depreciation

$$\delta_g = \delta_g^0 + \left(1 - \delta_g^0\right) \cdot \left(1 - ad_K^g\right) \cdot d_{TF,-1}$$ (138) Impact of climate change on green capital stock depreciation

$$a_f = a_{f,-1} \cdot \left(1 + g_f\right) \cdot \left[1 - (1 - ad_P) \cdot d_{TP,-1}\right]$$ (139) Product per unit of (either conventional or green) capital

$$g_f = g_{f0} + g_{f1} \cdot g_{BE,-1}$$ (140) Growth rate of product per unit of capital

$$a_n = a_{n,-1} \cdot (1 + g_n) \cdot \left[1 - (1 - ad_P) \cdot d_{TP,-1}\right]$$ (141) Labour productivity

$$g_n = g_{n0} + g_{n1} + g_{n2} \cdot g_{y,-1}$$ (142) Growth rate of labour productivity

$$g_{n0} = g_{n0,-1} \cdot (1 - g_{n3})$$ (143) Deceleration rate of labour productivity

$$d_T = 1 - \frac{1}{1 + dam_1 \cdot T_{AT} + dam_2 \cdot T_{AT}^2 + dam_3 \cdot T_{AT}^x}$$ (144) Proportion of gross damage due to changes in at. temperature ($x = 6.6754$)

$$d_{TP} = dam_P \cdot d_T$$ (145) Productivity damage

$$d_{TF} = 1 - \frac{1 - d_T}{1 - d_{TP}}$$ (146) Fund damage

Labour force, Employment and Working Time

$LF = LF_{-1} \cdot (1 + g_{LF}) \cdot \left[1 - (1 - ad_{LF}) \cdot d_{TF,-1}\right]$ (147) Labour force level

$g_{LF} = lf_0 + lf_1 - lf_2 \cdot un_{-1} - lf_3 \cdot hratio_{-1}$ (148) Labour force growth rate

$lf_0 = lf_{0,-1} \cdot (1 - lf_4)$ (149) Autonomous component of labour force growth rate

$N = \frac{Y_s}{H \cdot a_n}$ (150) Employment level

$H = H_{-1} + h_1 \cdot (em_{-1} - h_2)$ (151) Annual working time

$em = \frac{N}{LF}$ (152) Employment rate

$un = 1 - em$ (153) Unemployment rate

Production Function and Price Level

$Y_f^* = a_f \cdot K_{f,-1}$ (154) Capital-determined potential output

$Y_n^* = a_n \cdot LF_{-1} \cdot H_{-1}$ (155) Labour-determined potential output

$Y_m^* = \frac{k_{m,-1} + rec}{\mu}$ (156) Matter-determined potential output

$Y_{en}^* = \frac{k_{en,-1}}{\varepsilon}$ (157) Energy-determined potential output

$Y_{tec}^* = \min\left(Y_f^*, Y_n^*\right)$ (158) Economically-constrained potential output

$Y_{eco}^* = \min(Y_m^*, Y_{en}^*)$ (159) Ecologically-constrained potential output

$Y^* = \min(Y_{tec}^*, Y_{eco}^*)$ (160) Potential output (Leontief function)

$p_y = \frac{w}{a_n} \cdot (1 + mk)$ (161) Unit price of production

$w = w_{-1} \cdot \left(1 + w_a \cdot \frac{d(a_n)}{a_n}\right)$ (162) Money wage rate

$mk = mk_0 + mk_1 \cdot \frac{Y_{S,-1}}{Y_{-1}^*}$ (163) Gross mark-up over labour cost

$p = \pi_1 \cdot p_y + \pi_2 \cdot p_{en} + \pi_3 \cdot p_m$ (164) General price level (output deflator)

$p_w = p \cdot (1 + vat_w)$ (165) Price paid by LC including VAT

$p_\pi = p \cdot (1 + vat_\pi)$ (166) Price paid by UC including VAT

$E(x) = x_{-1} + \psi \cdot [E(x_{-1}) - x_{-1}]$ (167) Expectation function (with: $x = p, r_b, r_d, r_e$)

Other Utilisation Rates

$u_m = \frac{Y_s}{Y_m}$ (168) Matter utilisation rate

$u_{en} = \frac{Y_s}{Y_{en}}$ (169) Energy utilisation rate

Redundant Equation

$H_s = H_d$ Cash: supply = demand