Circular economy in a simplified input-output stock-flow consistent dynamic model

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Abstract

The aim of this paper is twofold. First, a simple input-output stock-flow consistent dynamic model is developed from scratch, in which *fiat* money is endogenously created, prices are defined in a Sraffa-like fashion, and markups depend on temporary output gaps. Second, the model is used to test the impact of a simple "circular economy" innovation on output and waste.

Keywords: Stock-Flow Consistent Models, Input-Output Analysis, Circular Economy

JEL Classification: E16, E17, C67, D57

Contents

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1 The simplest model

In the last two decades, stock-flow consistent (SFC) models have gained momentum in macroeconomics and ecological economics (e.g. Carnevali et al. 2019[2]). Arguably, the reason is that they are one of the most flexible and versatile tools to simulate, analyze and compare alternative policy scenarios. However, the main limitation of standard SFC models is that they only consider aggregate output. As a result, traditional SFC models do not allow accounting for the structural changes that incessantly take place in a capitalist economy. The aim of this work is to show how a basic input-output stock-flow consistent (IO-SFC) model can be developed and used to assess "circular economy" innovations and policies. For this purpose, the simplest aggregative SFC model (that is, "Model SIM" presented in Godley and Lavoie 2007, chapter 3[3]) is turned into an IO-SFC model in which *fiat* money is endogenously created, prices are defined in a Sraffa-like fashion, and markups depend on temporary output gaps.

Four macroeconomic sectors are considered: *a*) households; *b*) production firms; *c*) the government; and *d*) the central bank. Households are the recipients of every flow of income. They buy consumption goods based on both their disposable income and their stock of net wealth. Household savings are made up of government bills and/or cash.[1] There are two industries, in which firms produce two types of goods (and waste) by means of the same goods used as inputs. For the sake of simplicity, firms do not invest in fixed capital and they do not hold inventories. Firms make no net profit, as corporate incomes are entirely distributed to households in the form of wages and salaries. There are no commercial banks, hence no loans and no private money (bank deposits). The only type of money is cash (or *fiat* money), which is issued by the central bank as the government sector runs into budget deficits.

Let's start by defining households' total consumption in real terms (meaning at constant prices), which is:

$$c = \alpha_1 \cdot \frac{YD_{-1}}{p_{A,-1}} + \alpha_2 \cdot \frac{V_{-1}}{p_{A,-1}} \tag{1}$$

where p_A is a consumer price index, while α_1 and α_2 are the propensities to consume out of disposable income (YD) and net wealth (V), respectively.

Final demand is made up of both household consumption and government spending. For the sake of simplicity, we assume that there are only two industries/products. The final demand vector is:

$$\mathbf{d} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \mathbf{b} \cdot c + \mathbf{s} \cdot g = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \cdot c + \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} \cdot g \tag{2}$$

where $\mathbf{b}$ is the vector of household consumption shares (with: $\beta_2 = 1 - \beta_1$), g is total government consumption, and $\mathbf{s}$ is the vector of government consumption shares (with: $\sigma_2 = 1 - \sigma_1$).

The (gross) output vector is therefore:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{A} \cdot \mathbf{x} + \mathbf{d}$$

from which:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{d} \tag{3}$$

[1] Unlike the original Model SIM, and like Model PC (see Godley and Lavoie 2007, chapter 4[3]), Model IO-SIM explicitly considers portfolio choices of households.

where **I** is the identity matrix and **A** is the matrix of technical coefficients, defined as:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

As usual, a_{11} is the quantity of product 1 necessary to produce one unit of product 1, a_{12} is the quantity of product 2 necessary to produce one unit of product 1, and so on. Notice that the term $(\mathbf{I} - \mathbf{A})^{-1}$ is also a matrix. It is named the *Leontief inverse* and shows the multipliers, that is, the successive changes in production processes triggered by an initial change in consumption demands.

We can calculate nominal output as the product of the unit price vector and the output vector:

$$Y = \mathbf{p}^T \cdot \mathbf{x}$$

where the subscript "T" stands for the transpose of the matrix.

Similarly, the nominal net product of the economy is:

$$Y_n = \mathbf{p}^T \cdot \mathbf{d} \tag{4}$$

The employment level is determined by firms' demand for labour in each production process, that is:

$$N = \mathbf{x}^T \cdot \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} \oslash \mathbf{pr} \right] = \mathbf{x}^T \cdot \mathbf{l} \tag{5}$$

where $\oslash$ is the Hadamard division,[2] **pr** is the labour productivity vector, [3] and therefore **l** is the vector of labour coefficients.

We can either take the unit prices as exogenous variables and determine markups as endogenous variables or do the other way round. If we assume that production firms set the markups, then the unit price vector is:

$$\mathbf{p} = w \cdot \mathbf{l} + \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \mathbf{m} \right] \odot \mathbf{A} \cdot \mathbf{p} \tag{6}$$

hence:

$$\mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} \frac{w}{pr_1} + (1 + \mu_1) \cdot (p_1 \cdot a_{11} + p_2 \cdot a_{12}) \\ \frac{w}{pr_2} + (1 + \mu_2) \cdot (p_1 \cdot a_{21} + p_2 \cdot a_{22}) \end{pmatrix}$$

where **m** is the vector of markups and $\odot$ is the Hadamard product.[4] Like Sraffa, we assume that "the wage is paid *post factum* as a share of the annual product" (Sraffa 1960, p. 11[4]). However, the profit rate is allowed to differ across industries.

The price level faced by the households (consumption deflator or consumer price index) depends on the basket of goods they consume, which is:

$$p_A = \mathbf{p}^T \cdot \mathbf{b} \tag{7}$$

Similarly, the deflator for government spending is:

$$p_G = \mathbf{p}^T \cdot \mathbf{s} \tag{8}$$

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[2] Also called element-wise division of matrices.

[3] Notice that $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \oslash \mathbf{pr}$ is the vector of labour coefficients, that is: $\begin{pmatrix} 1/pr_1 \\ 1/pr_2 \end{pmatrix} = \mathbf{l}$.

[4] Also named element-wise multiplication of matrices.

Taxes collected by the government can be calculated using the average tax rate on households' total income. The total tax revenue is therefore:

$$T = \theta \cdot (Y_n + r_{-1} \cdot B_{h,-1}) \tag{9}$$

where θ is the average tax rate on income, r is the return rate on government bills, and B_h is the stock of bills held by the households.

Disposable income is nominal net income from firms *plus* (received) interest payments on government debt *minus* taxes:

$$YD = Y_n + r_{-1} \cdot B_{h,-1} - T \tag{10}$$

The government sector issues bills as it runs into budget deficits:

$$B_s = B_{s,-1} + g \cdot p_G - T \tag{11}$$

The central bank acts as a lender of last resort for the government sector:

$$B_{cb} = B_s - B_h \tag{12}$$

Cash is created as the central bank buys the bills that were not purchased by the private sector:

$$H_s = H_{s,-1} + (B_{cb} - B_{cb,-1}) \tag{13}$$

Cash is one of the two financial assets that the households can opt for. Net private wealth increases as households save:

$$V = V_{-1} + YD - c \cdot p_A \tag{14}$$

Household holdings of government bills are a proportion of their net wealth, which depends positively on the interest rate and negatively on the transactions demand for money (approximated by the disposable income to wealth ratio):

$$\frac{B_h}{V} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \frac{YD}{V} \tag{15}$$

The residual amount of net wealth is held in the form of cash:

$$H_h = V - B_h \tag{16}$$

Like in the original SIM model, the redundant equation is:

$$\Delta H_h = \Delta H_s$$

The model is now complete. We can name it *Model IO-SIM* (where IO stands for "Input-Output" and SIM stands for "Simplest SFC model"). In the next section, we check the accounting consistency and we assess its dynamic behaviour.

2 Model consistency and early experiments

Figure 1a shows that the hidden or redundant equation is always met. There are no accounting "black holes" and the model is fully consistent. The additional figures show how key model variables behave over time.

Like in Model SIM, the economy is set in motion by an initial expenditure from the government sector. Private firms produce both goods 1 and goods 2 on demand (Figure 1d). This generates an increase in output, disposable income, and consumption (Figure 1b,c,d). The economy grows following the initial shock and then stabilises at a new steady-state, where private consumption equals disposable income and the stock of net wealth remains unchanged (Figure 1e,f).

3 Incomplete adjustment

Equation (3) assumes that outputs fully and instantaneously adjusts to final demands. Arguably, this assumption is too strong and must be relaxed. A simple way to do that is to remind that the *Leontief inverse* can be expressed as a sum of power series (Waugh 1950[5]):

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\ \dots\ +\mathbf{A}^t+\ \dots\ = \sum_{t=0}^{\infty}\mathbf{A}^t$$

Therefore, an incomplete adjustment of production to demand conditions can be obtained using the following:

$$\mathbf{x} = \sum_{t=0}^{n}\mathbf{A}^t\cdot\mathbf{d} \tag{3B}$$

which only considers n rounds of adjustments of output to the initial change in demand. For instance, in the first round ($n=1$) the equation above becomes:

$$\mathbf{x} = (\mathbf{I}+\mathbf{A})\cdot\mathbf{d}$$

If subscript t is used as a time index and t_0 is the period in which the change demand occurs, we can re-write equation (3B) as:

$$\mathbf{x} = \sum_{t=t_0}^{n-t_0}\mathbf{A}^t\cdot\mathbf{d} \tag{3C}$$

If we replace equation (3) with equation (3C), the output vector adjusts gradually to demand shocks as the time goes by. In fact, we can rename $\mathbf{x}$ in equation (3) as $\mathbf{x}^*$, where the star stands for "fully adjusted". We can then use the gap between fully-adjusted outputs (as determined by equation 3) and current outputs (as determined by equation 3C) to endogenize industry markups:

$$\mathbf{m} = \mathbf{m}_0 + \mathbf{m}_1\odot(\mathbf{x}^*-\mathbf{x}) \tag{17}$$

where $\mathbf{m}_0 = (\mu_{10},\mu_{20})$ is the column vector of *normal* markups and $\mathbf{m}_1 = (\mu_{11},\mu_{12})$ is the column vector of markup elasticities to output gaps, so that:

$$\mathbf{m} = \begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix} = \begin{pmatrix}\mu_{10}+\mu_{11}\cdot(x_1^*-x_1)\\ \mu_{20}+\mu_{21}\cdot(x_2^*-x_2)\end{pmatrix}$$

Equation (12) holds that the markups increase above their normal rates as long as current outputs fall short of (fully-adjusted) demand-implied outputs. It captures demand pressures on supply conditions. Figure 2a shows output gaps in the two industries following the initial spending, while Figure 2b shows how unit prices react. Notice that the model can be easily extended to an n-industry economy. For instance, Figure 3a,b shows different expenditures and their impact on prices in a 3-industry economy.

4 Introducing the circular economy

The label "circular economy" (CE) denotes a set of policies and practices that aim at reusing, repairing, sharing, and recycling products and resources to create a closed-loop system, thus minimising waste, pollution and CO_2 emissions.[5] A simple way to introduce CE in the model above is to consider a 3-sector economy, in which the first two sectors produce *manufacturing* goods whereas the third sector is waste.

As long as waste is not recycled, the matrix of technical coefficients is:

[5] For a thorough discussion on the definition of CE, see Bimpizas-Pinis et al. 2021.[1]

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{pmatrix}$$

Production processes 1 and 2 generate waste, but no waste is reused for production. For the sake of simplicity, it is also assumed that idle waste does not generate additional waste. In this simplified model, CE can be introduced through a change in technical coefficients such that the new matrix is:

$$\mathbf{B} = \begin{pmatrix} b_{11} \leq a_{11} & b_{12} \leq a_{12} & b_{13} \geq 0 \\ b_{21} \leq a_{21} & b_{22} \leq a_{22} & b_{23} \geq 0 \\ b_{31} \geq a_{31} & b_{32} \geq a_{32} & 0 \end{pmatrix}$$

In short, CE entails a reduction in one or more coefficients defining the quantities of product 1 and product 2 used as inputs (green coefficients). This is possible because waste now enters the production of outputs 1 and 2 (blue coefficients).[6] Besides, the increase in waste efficiency entails an increase in the related technical coefficients (red coefficients) compared with the baseline scenario.

Figure 4 shows that a CE innovation is associated with a reduction in production costs, hence prices. This, in turn, may well enhance consumption, thus increasing demand and real production. Arguably, this is the most likely scenario under a free-market economy. Efficiency increases, but the growth in consumer demand fully offsets the ecological benefits (namely, the reduction in inputs and waste per unit of output) that are brought about by the CE innovation.[7] Notice, however, that this potential gain could be turned into an actual gain (that is, into a reduction in waste production) if the government stabilized consumption and output through (direct or indirect) taxation. In fact, the related tax revenue could be employed to fund and support CE practices.

5 Final remarks

Simplified though it is, Model IO-SIM provides an intuitive, but sound, base for developing more sophisticated IO-SFC dynamic models. A variety of sectors, industries, products and financial assets, as well as ecosystem-related variables, can be easily factored in. Unlike standard SFC models, Model IO-SIM allows dealing with structural changes. Unlike traditional IO models, it enables one to endogenize and link the changes in technical coefficients with the evolution of demand conditions, the financial sector and the broad ecosystem. As a result, a variety of feedback effects can be explicitly modeled. Looking at future developments and empirical applications, the main issue seems to be the consistency and availability of data.

[6] Therefore, CE implies a degree of substitutability of inputs. Notice that while *full input substitutability* is one of the key assumptions of mainstream models, it has been harshly criticized by several dissenting economists.

[7] This *rebound effect* is a well-known issue, which does not depend on the (low) degree of sophistication of the model – see Zink and Geyer 2017.[6], and Bimpizas-Pinis et al. 2021.[1]

References

[1] Bimpizas-Pinis, M., Bozhinovska, E., Genovese, A., Lowe, B., Pansera, M., Alberich, J.P. and Ramezankhani, M.J. (2021) Is efficiency enough for circular economy?. *Resources, Conservation & Recycling*, 167: 105399.

[2] Carnevali, E., Deleidi, M., Pariboni, R. and Veronese Passarella, M. (2019) SFC dynamic models: features, limitations and developments, in P. Arestis and M. Sawyer (eds.), *Frontiers of Heterodox Economics, Series: International Papers in Political Economy*, Basingstoke & New York: Palgrave Macmillan, pp. 223-276.

[3] Godley, W. and Lavoie, M. (2007) *Monetary economics: an integrated approach to credit, money, income, production and wealth.* Springer.

[4] Sraffa, P. (1960) *Production of commodities by means of commodities. Prelude to a critique of economic theory.* Vora & co.

[5] Waugh, F.V. (1950) Inversion of the Leontief matrix by power series. *Econometrica*, 18(2): 142-154.

[6] Zink, T. and Geyer, R. (2017) Circular Economy Rebound. *Journal of Industrial Ecology*, 21: 593-602.

A Figures and tables

Figure 1: Model consistency and baseline

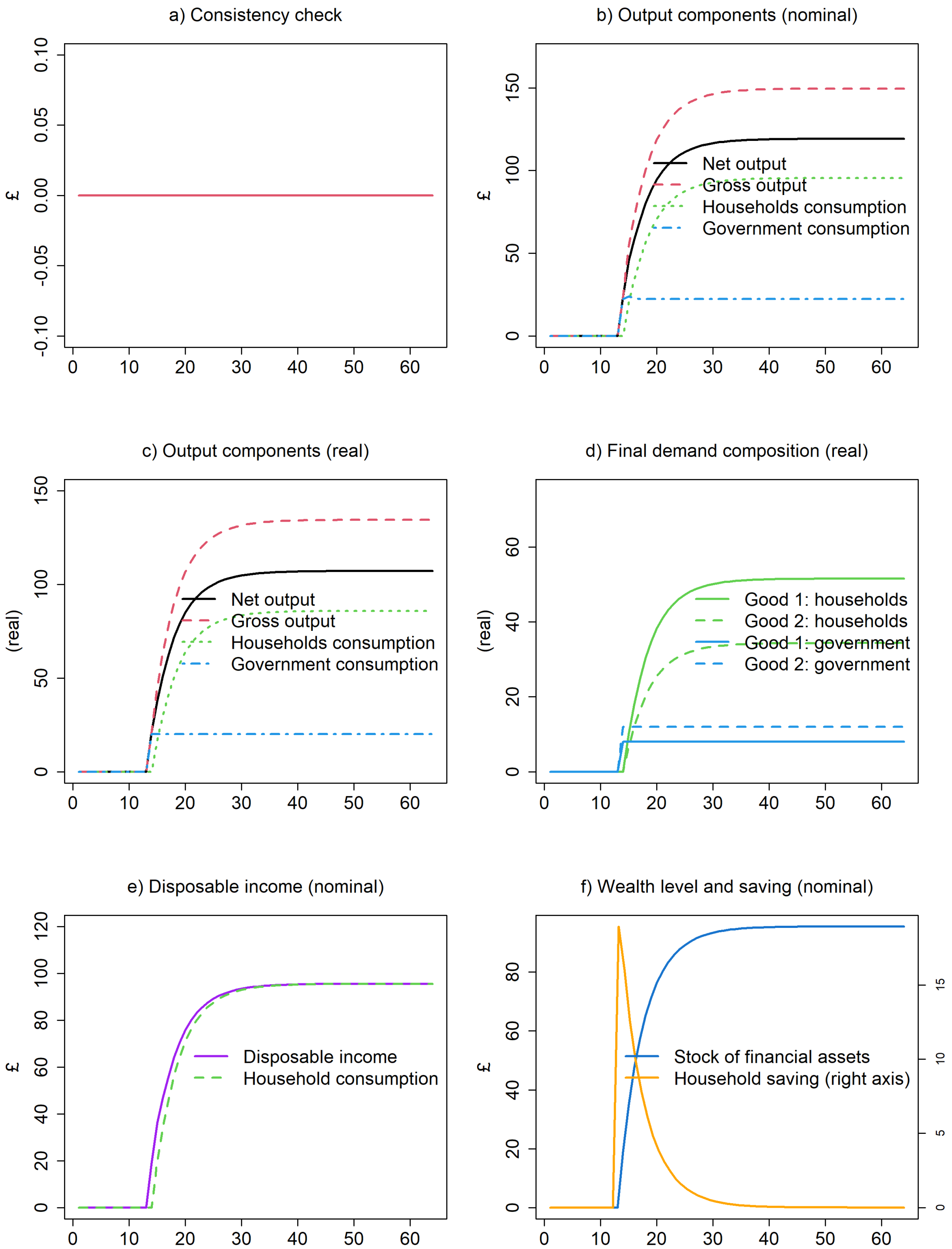

Figure 2: Output adjustment following a shock to demand

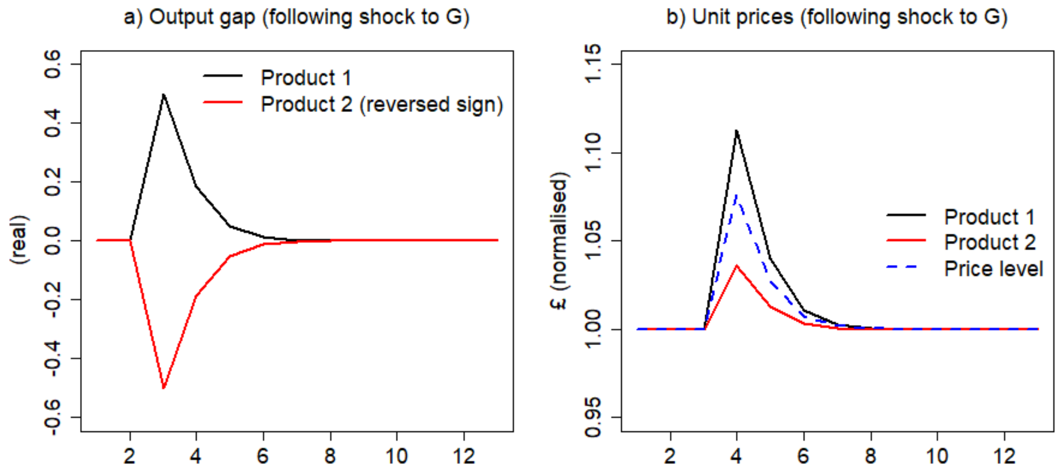

Figure 3: Demands and prices in a 3-industry model

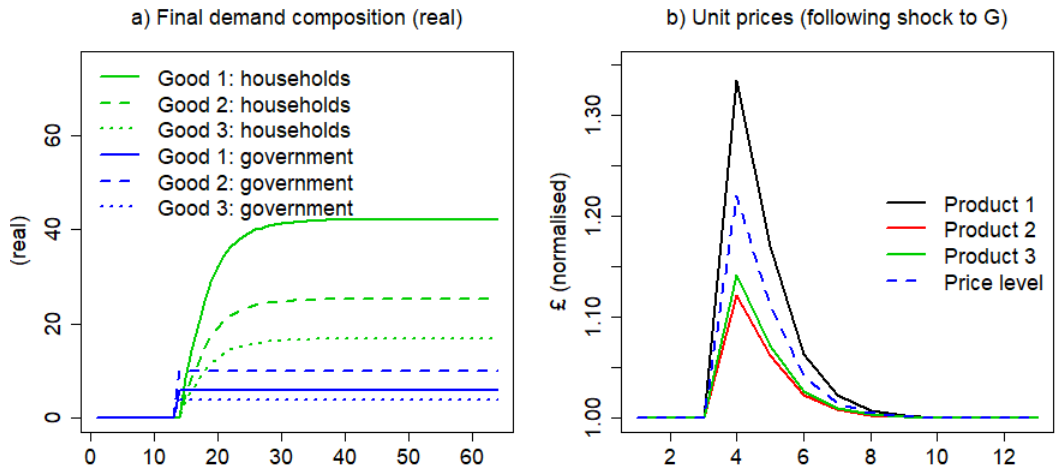

Figure 4: CE policy in a "2-industry + waste" model

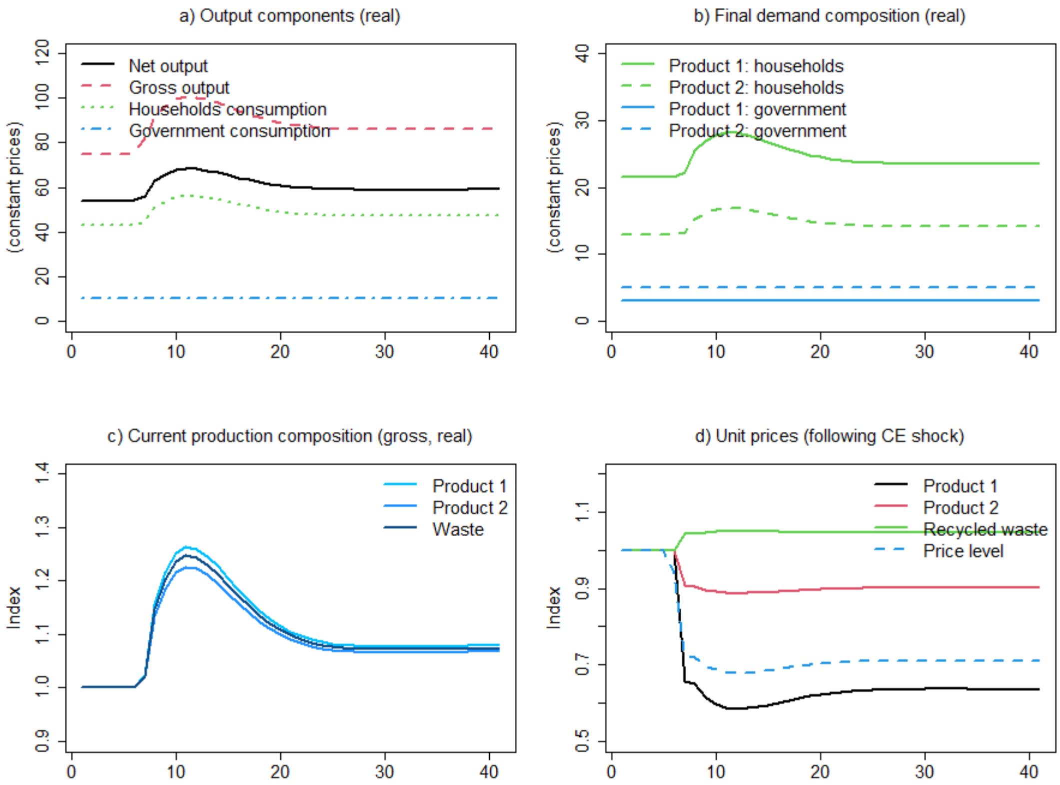

Figure 5: Transactions-flows and changes in nominal wealth

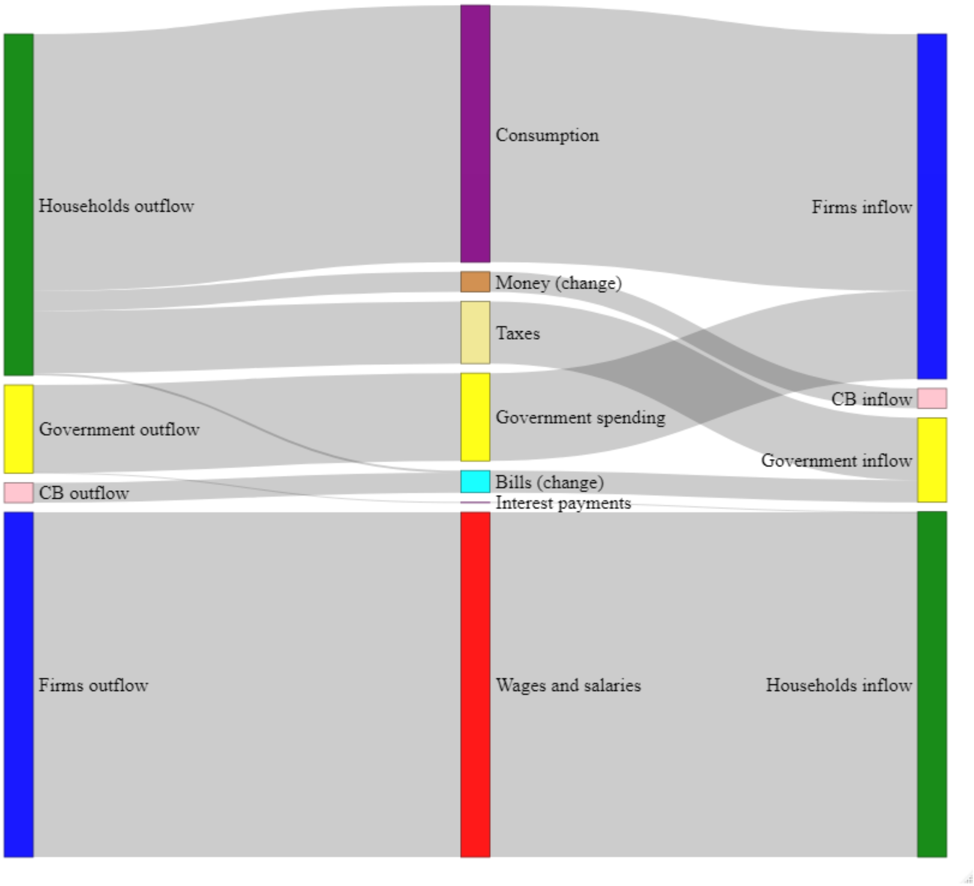

Figure 6: Production structure without CE

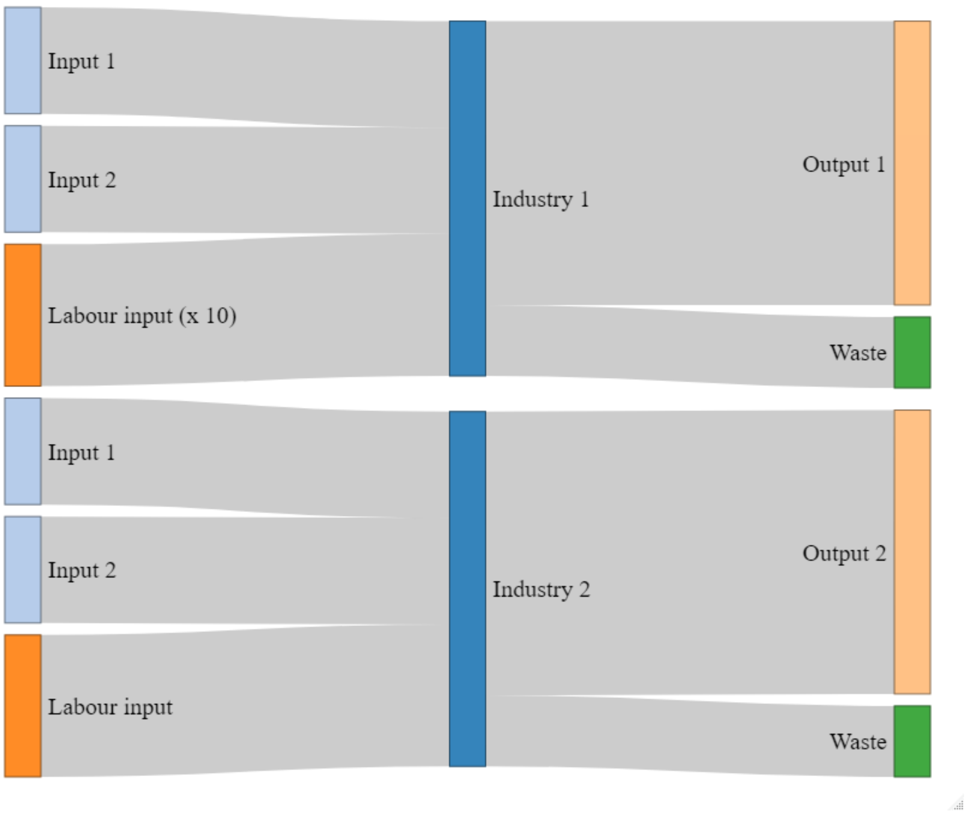

Figure 7: Production structure with CE

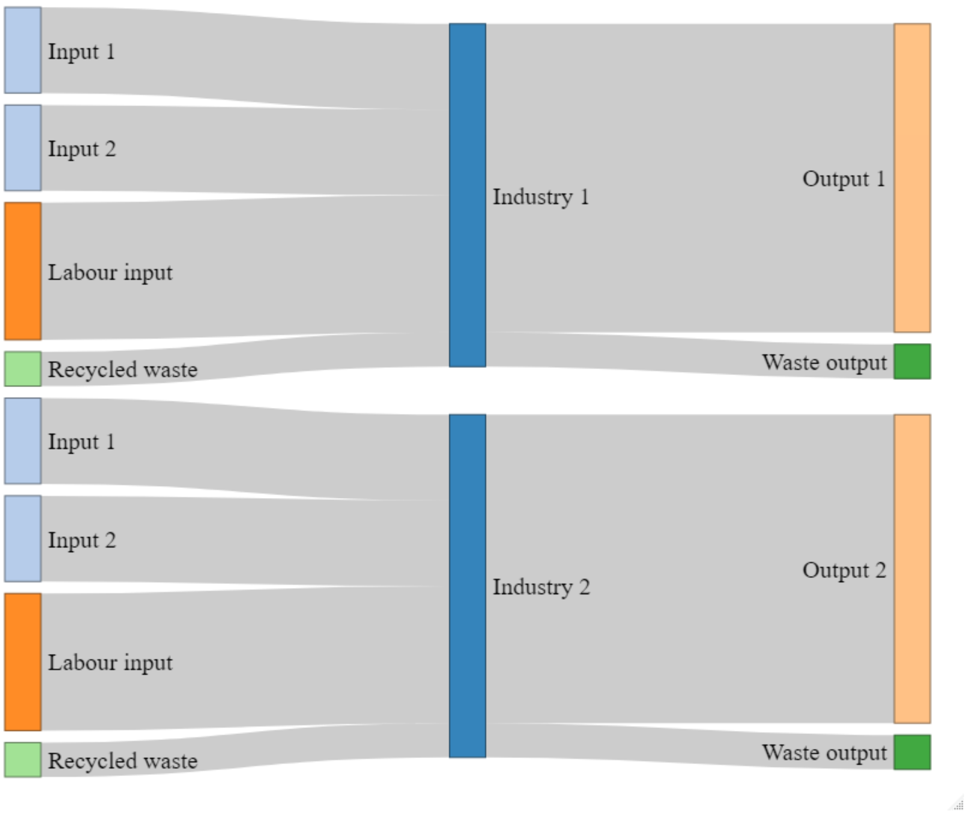

Table 1: Table 1. Balance sheet in $t = 20$ (baseline scenario)

	Households	Firms	Government	Central Bank	Total
Money	25.65	0	0.0	-25.65	0
Bills	2.85	0	-28.5	25.65	0
Net financial wealth	-28.50	0	28.5	0.00	0
Total	0.00	0	0.0	0.00	0

Table 2: Table 2. Transactions-flow matrix in $t = 20$ (baseline scenario)

	Households	Firms	Government	Central Bank	Total
Consumption	-25.86	25.86	0.00	0.00	0
Government spending	0.00	9.55	-9.55	0.00	0
Output	0.00	[35.41]	0.00	0.00	0
Wages and salaries	35.41	-35.41	0.00	0.00	0
Taxes	-7.08	0	7.08	0.00	0
Interest payments	0.05	0	-0.05	0.00	0
Change in money stock	-2.27	0	0.00	2.27	0
Change in bills	-0.25	0	2.52	-2.27	0
Total	0.00	0	0.00	0.00	0

B R code of Model IO-SIM (2 industries)

```
#UPLOAD LIBRARIES
library(expm)

#CLEAR
rm(list=ls(all=TRUE))

#PERIODS (i= 1 to 65)
nPeriods=65

#PARAMETERS
alpha1 = 0.6   #Propensity to consume out of income
alpha2 = 0.4   #Propensity to consume out of wealth
theta = 0.2    #Tax rate
mu10 = 0.1     #Autonomous component of markup on price of product 1
mu11 = 0.75    #Output-gap elasticity of price of product 1
mu20 = 0.1     #Autonomous component of markup on price of product 2
mu21 = 0.25    #Output-gap elasticity of price of product 2
r = 0.02       #Interest rate
lambda0 = 0.1 #Fixed proportion of government bills to total wealth
lambda1 = 0    #Sensitivity of portfolio choices to interest rate
lambda2 = 0    #Sensitivity of portfolio choices to transactions demand for money

#VARIABLES
#Gross product
y=matrix(data=0,nrow=1,ncol=nPeriods)
#Net product
yn=matrix(data=0,nrow=1,ncol=nPeriods)
#Total consumption
c=matrix(data=0,nrow=1,ncol=nPeriods)
#Government expenditures
g=matrix(data=0,nrow=1,ncol=nPeriods)
#Taxes
t=matrix(data=0,nrow=1,ncol=nPeriods)
#Disposable income
yd=matrix(data=0,nrow=1,ncol=nPeriods)
#Cash demand
h_h=matrix(data=0,nrow=1,ncol=nPeriods)
#Cash supply
h_s=matrix(data=0,nrow=1,ncol=nPeriods)
#Supply of bills
b_s=matrix(data=0,nrow=1,ncol=nPeriods)
#Private demand for bills
b_h=matrix(data=0,nrow=1,ncol=nPeriods)
#CB holdings of bills
b_cb=matrix(data=0,nrow=1,ncol=nPeriods)
#Net wealth of households
v=matrix(data=0,nrow=1,ncol=nPeriods)
#Labour
n=matrix(data=0,nrow=1,ncol=nPeriods)
#Product per worker
pr=matrix(data=c(1.2,0.8),nrow=2,ncol=nPeriods)
#Additional matrix to calculate labour coefficients
I2=matrix(data=c(1,1),nrow=2,ncol=nPeriods)
```

```
#Wage rate
w=matrix(data=0.86,nrow=1,ncol=nPeriods)
#Vector of markup
mu=matrix(data=c(mu10,mu20),nrow=2,ncol=nPeriods)
#Matrix of coefficients
A=matrix(data=c(0.1,0.1,0.1,0.1),nrow=2,ncol=2)
#Auxiliary matrix of coefficients
B=matrix(data=c(0,0,0,0),nrow=2,ncol=2)
#Identity matrix
I <- diag(2)
#Production vector
x=matrix(data=0,nrow=2,ncol=nPeriods)
#Fully-adjusted production vector
x_star=matrix(data=0,nrow=2,ncol=nPeriods)
#Final demand vector
d=matrix(data=0,nrow=2,ncol=nPeriods)
#Price vector
p=matrix(data=c(0.9,1.1),nrow=2,ncol=nPeriods)
#Share of government consumption
sigma=matrix(data=c(0.4,0.6),nrow=2,ncol=nPeriods)
#Share of households consumption
beta=matrix(data=c(0.6,0.4),nrow=2,ncol=nPeriods)
#General price level
pa=matrix(data=0,nrow=1,ncol=nPeriods)
#Government spending deflator
pg=matrix(data=0,nrow=1,ncol=nPeriods)

#MODEL

#Define time loop
for (i in 2:nPeriods){

  #Define iterations loop (to force convergence to simultaneous solution)
  for (iterations in 1:20){

    #Introduce shock to government spending
    if (i>=15){g[1,i]=20} #Government expenditures passes
                                    #from 0 to 20 after 15 periods

    #Create the model (system of difference equations)

    #Total consumption
    if (i<=2) {c[,i] = alpha1*yd[,i-1] + alpha2*v[,i-1]}
    else{c[,i] = alpha1*yd[,i-1]/pa[,i-1] + alpha2*v[,i-1]/pa[,i-1]}

    #Final demand/consumption vector
    d[,i] = beta[,i]*c[,i] + sigma[,i]*g[,i]

    #Gross production vector (full adjustment)
    x_star[,i] = solve(I-A) %*% d[,i]

    #Gross product in nominal terms
    y[,i] = t(p[,i]) %*% (I+B) %*% d[,i] #t(p[,i]) %*% x[,i]

    #Net product in nominal terms
    yn[,i] = t(p[,i]) %*% d[,i]
```

```
    #Endogenous price vector (using Hadamard product)
    p[1,i] = (p[2,i]*A[3]*(1+mu[1,i]) + w[,i]/pr[1,i])/(1 - A[1]*(1+mu[1,i]))
    p[2,i] = (p[1,i]*A[2]*(1+mu[2,i]) + w[,i]/pr[2,i])/(1 - A[4]*(1+mu[2,i]))

    #Endogenous markup
    mu[1,i] = mu10 + mu11*(x_star[1,i-1]-x[1,i-1])
    mu[2,i] = mu20 + mu21*(x_star[2,i-1]-x[2,i-1])

    #General price level faced by households and firms
    pa[,i] = t(p[,i]) %*% beta[,i]

    #Deflator of government spending
    pg[,i] = t(p[,i]) %*% sigma[,i]

    #Tax payments
    t[,i] = theta*(yn[,i] + r * b_h[,i-1])

    #Disposable income
    yd[,i] = yn[,i] + r * b_h[,i-1] - t[,i]

    #Supply of government bills
    b_s[,i] = b_s[,i-1] + g[,i]*pg[,i] - t[,i] + r * b_h[,i-1]

    #CB holdings of bills
    b_cb[,i] = b_s[,i] - b_h[,i]

    #Supply of cash money
    h_s[,i] = h_s[,i-1] + (b_cb[,i] - b_cb[,i-1])

    #Net wealth of households
    v[,i] = v[,i-1] + yd[,i] - c[,i]*pa[,i]

    #Private demand for bills
    b_h[,i] = lambda0 * v[,i] + lambda1 * r * v[,i] + lambda2 * yd[,i]

    #Cash held by households
    h_h[,i] = v[,i] - b_h[,i]

  }

  #OUT-OF-ITERATION CALCULATIONS

  #Gross production vector (adjustment over time)
  if(i>=15){B = B + A%^%(i-14)}
  else{B = c(0,0,0,0) }
  x[,i] = (I+B) %*% d[,i]

        #Employment
  n[,i] = (I2[,i]/pr[,i]) %*% x[,i]

}

#PLOT CONSISTENCY CHECK
plot(h_h[1,2:65]-h_s[1,2:65],type="l",col=2,ylim=range(-0.1,0.1))
```