Circular economy in a simplified input-output stock-flow consistent dynamic model

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Abstract

The aim of this paper is twofold. First, Godley and Lavoie (2007[3])'s Model SIM(EX) is turned into an input-output stock-flow consistent model, where *fiat* money is endogenously created, prices are defined in a Sraffa-like fashion, and mark-up rates depend on temporary output gaps. Second, the model is used to test the impact of a (simplified) "circular economy" practice.

Keywords: Stock-Flow Consistent Models, Input-Output Analysis, Circular Economy

JEL Classification: E16, E17, C67, D57

Contents

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1 The simplest model

In the last two decades, stock-flow consistent (SFC) models have gained momentum in macroeconomics and ecological economics (e.g. Carnevali et al. 2019[2]). They are one of the most flexible and versatile tools to simulate, analyse and compare alternative scenarios. Arguably, the main limitation of standard SFC models is that they only consider aggregate production. As a result, they do not allow accounting for the structural changes of the economy. The aim of this work is to show how a simplified input-output (IO) SFC model can be developed and used to assess "circular economy" practices. For this purpose, Godley and Lavoie (2007[3])'s Model SIM(EX) is turned into an IO-SFC model in which *fiat* money is endogenously created, prices are defined in a Sraffa-like fashion, and mark-up rates depend on temporary output gaps.

Let's start by defining households' total consumption in real terms, which is:

$$c = \alpha_1 \cdot \frac{YD_{-1}}{p_{A,-1}} + \alpha_2 \cdot \frac{H_{d,-1}}{p_{A,-1}} \tag{1}$$

where p_A is the average price level, whereas α_1 and α_2 are the propensities to consume out of disposable income (YD) and net wealth (H_d), respectively.

Final demand is made up of both household consumption and government spending. For the sake of simplicity, we assume that there are only two products. The final demand vector (in real terms) is:

$$\mathbf{d} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \mathbf{b} \cdot c + \mathbf{s} \cdot g = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \cdot c + \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} \cdot g \tag{2}$$

where c is total household consumption, $\mathbf{b}$ is the vector of household consumption shares (with: $\beta_2 = 1 - \beta_1$), g is total government total consumption, and $\mathbf{s}$ is the vector of government consumption shares (with: $\sigma_2 = 1 - \sigma_1$).

The (gross) output vector is therefore:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{A} \cdot \mathbf{x} + \mathbf{d}$$

from which:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{d} \tag{3}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{A}$ is the matrix of technical coefficients, defined as:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

As usual, a_{11} is the quantity of product 1 necessary to produce one unit of product 1, a_{12} is the quantity of product 2 necessary to produce one unit of product 1, and so on. Notice that the term $(\mathbf{I} - \mathbf{A})^{-1}$ is also a matrix. It is named the *Leontief inverse* and shows the multipliers, that is, the successive changes in production processes triggered by an initial change in consumption demands.

We can calculate nominal output as the product of the unit price vector and the output vector:

$$Y = \mathbf{p}^T \cdot \mathbf{x}$$

where the subscript "T" stands for the transpose of the matrix.

Similarly, the nominal net product of the economy is:

$$Y_n = \mathbf{p}^T \cdot \mathbf{d} \tag{4}$$

The employment level is determined by firms' demand for labour in each production process, that is:

$$N = \mathbf{x}^T \cdot \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} \oslash \mathbf{pr} \right] = \mathbf{x}^T \cdot \mathbf{l} \tag{5}$$

where $\oslash$ is the Hadamard division,[1] $\mathbf{pr}$ is the labour productivity vector, [2] and therefore $\mathbf{l}$ is the vector of labour coefficients.

We can either set unit prices as exogenous variables and determine mark-up percentages as endogenous variables or do the other way round. If we assume that firms are price setters, meaning that they are able to set the mark-up, then the price vector is:

$$\mathbf{p} = w \cdot \mathbf{l} + \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \mathbf{m} \right] \odot \mathbf{A} \cdot \mathbf{p} \tag{6}$$

hence:

$$\mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} \frac{w}{pr_1} + (1+\mu_1) \cdot (p_1 \cdot a_{11} + p_2 \cdot a_{12}) \\ \frac{w}{pr_2} + (1+\mu_2) \cdot (p_1 \cdot a_{21} + p_2 \cdot a_{22}) \end{pmatrix}$$

where $\mathbf{m}$ is the vector of mark-ups and $\odot$ is the Hadamard product.[3] Like Sraffa, we assume that "the wage is paid *post factum* as a share of the annual product" (Sraffa 1960, p. 11[4]). However, the profit rate is allowed to differ across industries.

The average price level depends on the basket of goods consumed by households, which is:

$$p_A = \mathbf{p}^T \cdot \mathbf{b} \tag{7}$$

Total tax revenue received by the government is:

$$T = \theta \cdot Y_n \tag{8}$$

where θ is the average tax rate on net income.

Disposable income is nominal net income minus taxes:

$$YD = Y_n - T \tag{9}$$

Cash is created by the central bank as the government runs into budget deficits to buy products from the private sector:

$$H_s = H_{s,-1} + \mathbf{p}^T \cdot \mathbf{s} \cdot g - T \tag{10}$$

Hoseholds' cash holdings are:

$$H_h = H_{h,-1} + YD - c \cdot p_A \tag{11}$$

Like in the original SIM model, the redundant equation is:

$$H_d = H_h$$

The model is now complete. We can name it *Model IO-SIM* (where IO stands for "Input-Output" and SIM stands for "Simplest SFC model"). In the next section, we check the accounting consistency and we assess its dynamic behaviour.

[1] Also called element-wise division of matrices.

[2] Notice that $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \oslash \mathbf{pr}$ is the vector of labour coefficients, that is: $\begin{pmatrix} 1/pr_1 \\ 1/pr_2 \end{pmatrix} = \mathbf{l}$.

[3] Also named element-wise multiplication of matrices.

2 Model consistency and early experiments

Figure 1a shows that the hidden or redundant equation is always met. There are no accounting "black holes" and the model is fully consistent. The additional figures show how key model variables behave over time.

Like in Model SIM, the economy is set in motion by an initial expenditure from the government sector. Private firms produce both goods 1 and goods 2 on demand (Figure 1d). This generates an increase in output, disposable income, and consumption (Figure 1b,c,d). The economy grows following the initial shock and then stabilises at a new steady-state, where private consumption equals disposable income and the stock of net wealth remains unchanged (Figure 1e,f).

3 Incomplete adjustment

Equation (3) assumes that output fully and instantaneously adjusts to final demands. Arguably, this assumption is too strong and must be relaxed. A simple way to do that is to remind that the *Leontief inverse* can be expressed as a sum of power series (Waugh 1950[5]):

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\ \dots\ +\mathbf{A}^t+\ \dots\ = \sum_{t=0}^{\infty}\mathbf{A}^t$$

Therefore, an incomplete adjustment of production to demand conditions can be obtained using the following:

$$\mathbf{x} = \sum_{t=0}^{n}\mathbf{A}^t\cdot\mathbf{d} \tag{3B}$$

which only considers n rounds of adjustments of output to the initial change in demand. For instance, in the first round ($n=1$) the equation above becomes:

$$\mathbf{x} = (\mathbf{I}+\mathbf{A})\cdot\mathbf{d}$$

If subscript t is used as a time index and t_0 is the period where the demand changes, we can re-write equation (3B) as:

$$\mathbf{x} = \sum_{t=t_0}^{n-t_0}\mathbf{A}^t\cdot\mathbf{d} \tag{3C}$$

If we replace equation (3) with equation (3C), the output vector adjusts gradually to demand shocks as the time goes by. In fact, we can rename $\mathbf{x}$ in equation (3) as $\mathbf{x}^*$, where the star stands for "fully adjusted". We can then use the gap between fully-adjusted outputs (as determined by equation 3) and current outputs (as determined by equation 3C) to endogenise industry mark-ups:

$$\mathbf{m} = \mathbf{m}_0 + \mathbf{m}_1 \odot (\mathbf{x}^* - \mathbf{x}) \tag{12}$$

where $\mathbf{m}_0 = (\mu_{10}, \mu_{20})$ is the column vector of *normal* mark-up rates and $\mathbf{m}_1 = (\mu_{11}, \mu_{12})$ is the column vector of mark-up elasticities to output gaps, so that:

$$\mathbf{m} = \begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix} = \begin{pmatrix}\mu_{10}+\mu_{11}\cdot(x_1^*-x_1)\\ \mu_{20}+\mu_{21}\cdot(x_2^*-x_2)\end{pmatrix}$$

Equation (12) holds that mark-ups increase above their normal rates as long as current outputs fall short of (fully-adjusted) demand-implied outputs. It captures demand pressures on supply conditions. Figure 2b shows output gaps in the two industries following the initial spending, while Figure 2b shows how prices react. Notice that the model can be easily extended to an n-industry economy. For instance, Figure 3a,b shows different expenditures and their impact on prices in a 3-industry economy.

4 Introducing the circular economy

The label "circular economy" (CE) denotes a set of policies and practices that aim at reusing, repairing, sharing, and recycling products and resources to create a closed-loop system, thus minimising waste, pollution and CO_2 emissions.[4] A simple way to introduce CE in the model above is to consider a 3-sector economy, in which the first two sectors produce *manufacturing* goods whereas the third sector is waste. If waste is not recycled, the matrix of technical coefficients is:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{pmatrix}$$

Production processes 1 and 2 generate waste, but no waste is reused for production. For the sake of simplicity, it is also assumed that idle waste does not generate additional waste. In this simplified model, CE can be introduced through a change in technical coefficients such that the new matrix is:

$$\mathbf{B} = \begin{pmatrix} b_{11} \leq a_{11} & b_{12} \leq a_{12} & b_{13} \geq 0 \\ b_{21} \leq a_{21} & b_{22} \leq a_{22} & b_{23} \geq 0 \\ b_{31} \geq a_{31} & b_{32} \geq a_{32} & 0 \end{pmatrix}$$

In short, CE entails a reduction in one or more coefficients defining the quantities of product 1 and product 2 used as inputs (green coefficients). This is possible because waste now enters the production of 1 and 2 (blue coefficients).[5] Besides, the increase in waste efficiency entails an increase in the related technical coefficients (red coefficients) compared with the baseline scenario.

Figure 4 shows that CE allows reducing production costs, hence prices. This, in turn, may well enhance consumption, thus increasing demand and real production. Arguably, this is the most likely scenario under a market economy. Efficiency increases, but the growth in consumer demand fully offsets the ecological benefits (i.e. reduction in inputs and waste per unit of output here) of CE practices.[6] However, this potential gain can be turned into a reduction in waste level if the government stabilises consumption and output through direct or indirect taxation. In fact, the related tax revenue can be employed to fund and enhance CE practices.

[4] For a thorough discussion on the definition of CE, see Bimpizas-Pinis et al. 2021.[1]

[5] Therefore, CE implies a degree of substitutability of inputs. Notice that while *full input substitutability* is one of the key assumptions of mainstream models, it has been harshly criticised by several dissenting economists.

[6] This is a well-known issue, which does not depend on the (low) degree of sophistication of the model – see Zink and Geyer 2017.[6], and Bimpizas-Pinis et al. 2021.[1]

References

[1] Bimpizas-Pinis, M., Bozhinovska, E., Genovese, A., Lowe, B., Pansera, M., Alberich, J.P. and Ramezankhani, M.J. (2021) Is efficiency enough for circular economy?. *Resources, Conservation & Recycling*, 167: 105399.

[2] Carnevali, E., Deleidi, M., Pariboni, R. and Veronese Passarella, M. (2019) SFC dynamic models: features, limitations and developments, in P. Arestis and M. Sawyer (eds.), *Frontiers of Heterodox Economics, Series: International Papers in Political Economy*, Basingstoke & New York: Palgrave Macmillan, pp. 223-276.

[3] Godley, W. and Lavoie, M. (2007) *Monetary economics: an integrated approach to credit, money, income, production and wealth.* Springer.

[4] Sraffa, P. (1960) *Production of commodities by means of commodities. Prelude to a critique of economic theory.* Vora & co.

[5] Waugh, F.V. (1950) Inversion of the Leontief matrix by power series. *Econometrica*, 18(2): 142-154.

[6] Zink, T. and Geyer, R. (2017) Circular Economy Rebound. *Journal of Industrial Ecology*, 21: 593-602.

A Figures and tables

Figure 1: Model consistency and baseline

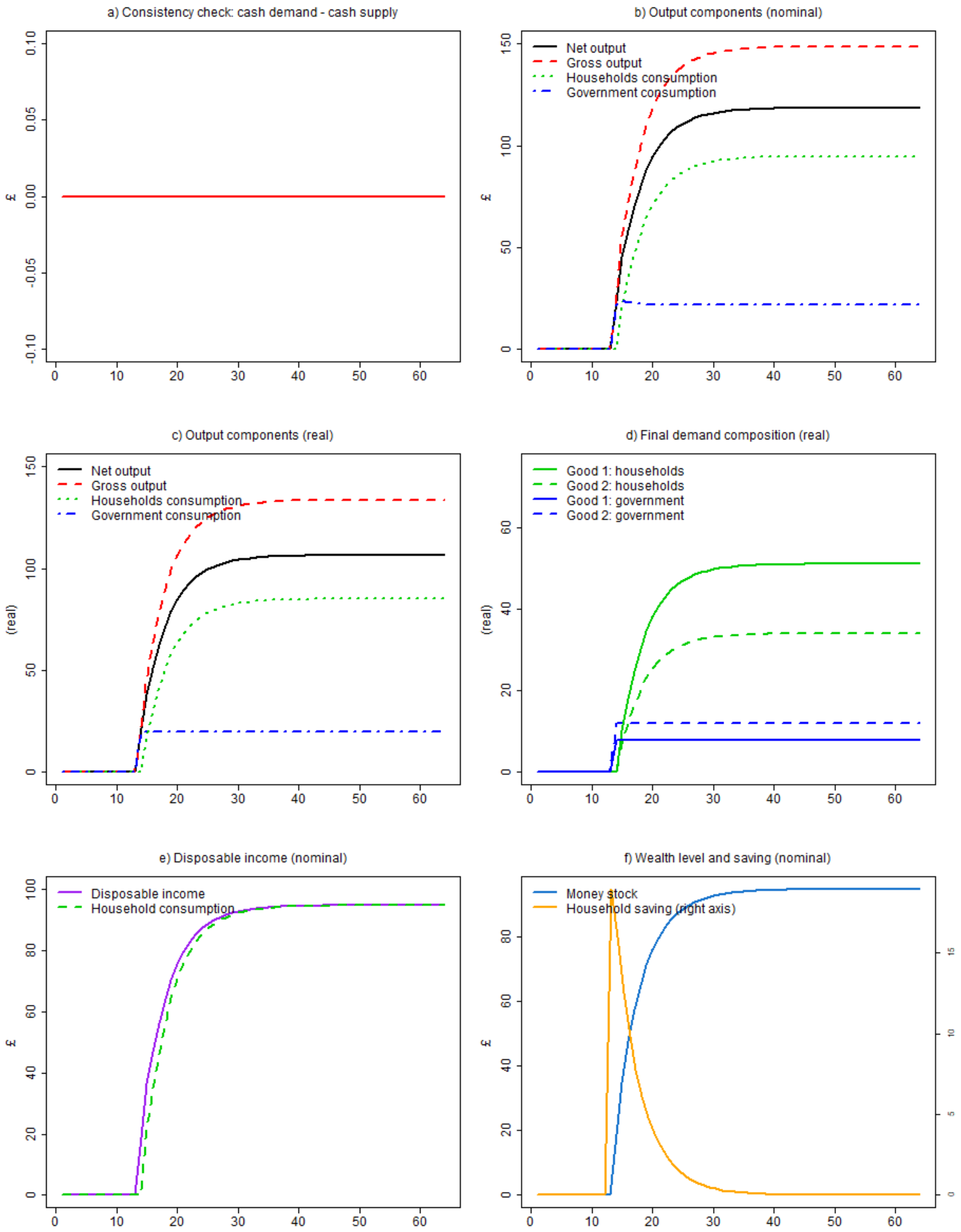


Figure 2: Output adjustment following a shock to demand

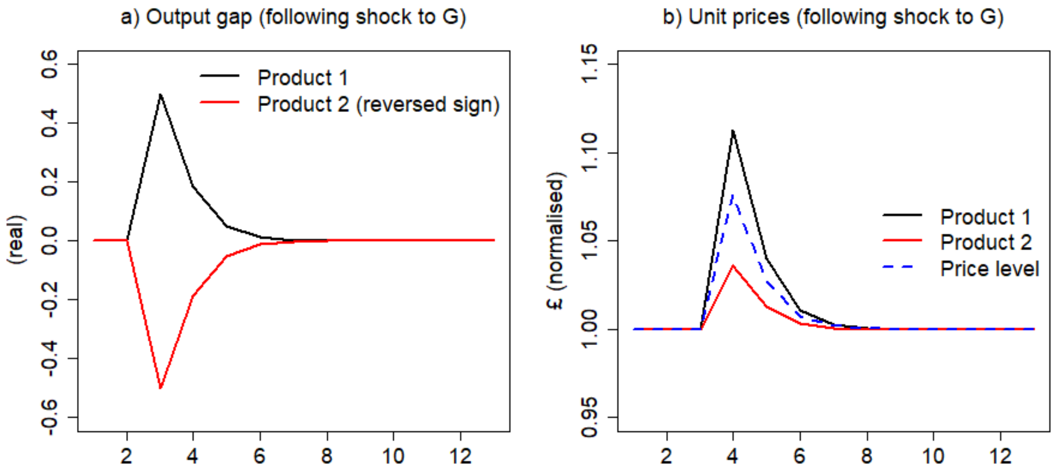


Figure 3: Demands and prices in a 3-industry model

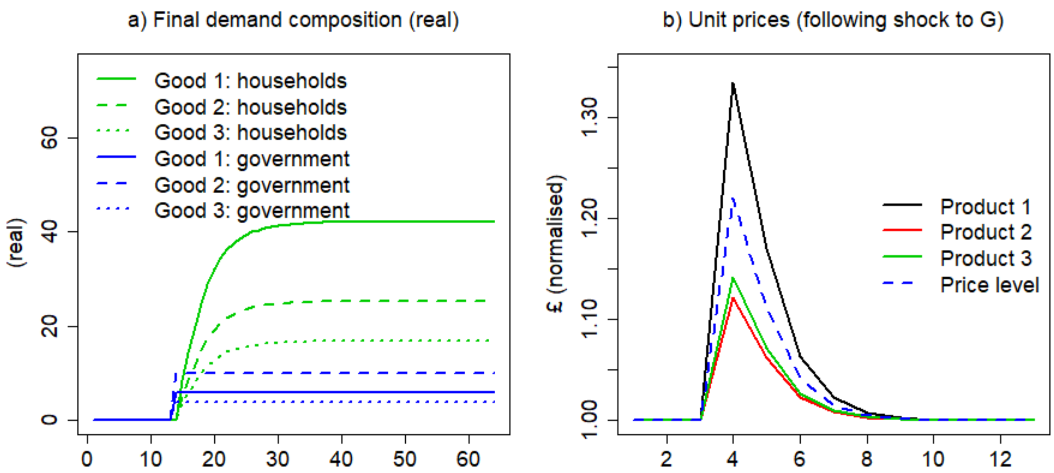
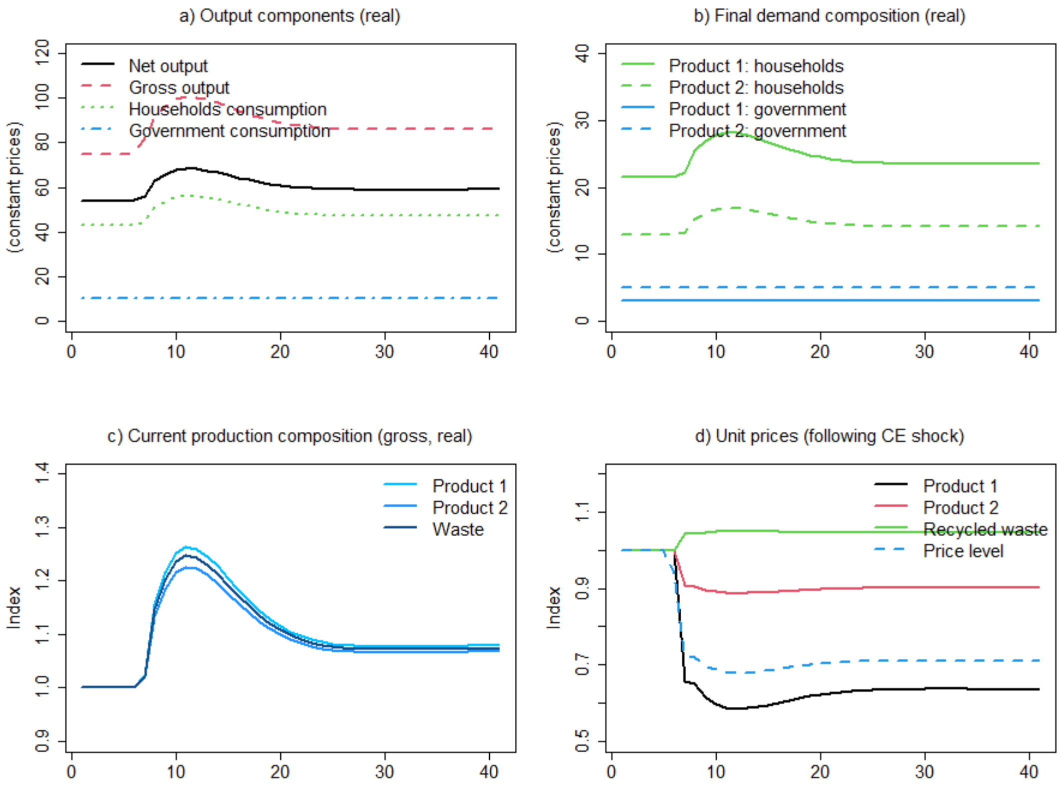


Figure 4: CE practices in a 3-industry model



B R code of Model IO-SIM (2 industries)

```
#UPLOAD LIBRARIES
library(expm)

#CLEAR
rm(list=ls(all=TRUE))

#PERIODS (i= 1 to 65)
nPeriods=65

#PARAMETERS
alpha1=0.6
alpha2=0.4
theta=0.2
mu10 = 0.1
mu11 = 0.75
mu20 = 0.1
mu21 = 0.25

#VARIABLES
#Gross product
y=matrix(data=0,nrow=1,ncol=nPeriods)
#Net product
yn=matrix(data=0,nrow=1,ncol=nPeriods)
#Total consumption
c=matrix(data=0,nrow=1,ncol=nPeriods)
#Government expenditures
g=matrix(data=0,nrow=1,ncol=nPeriods)
#Taxes
t=matrix(data=0,nrow=1,ncol=nPeriods)
#Disposable income
yd=matrix(data=0,nrow=1,ncol=nPeriods)
#Cash demand
h_h=matrix(data=0,nrow=1,ncol=nPeriods)
#Cash supply
h_s=matrix(data=0,nrow=1,ncol=nPeriods)
#Labour
n=matrix(data=0,nrow=1,ncol=nPeriods)
#Product per worker
pr=matrix(data=c(1.2,0.8),nrow=2,ncol=nPeriods)
#Additional matrix to calculate labour coefficients
I2=matrix(data=c(1,1),nrow=2,ncol=nPeriods)
#Wage rate
w=matrix(data=0.86,nrow=1,ncol=nPeriods)
#Vector of mark-up
mu=matrix(data=c(mu10,mu20),nrow=2,ncol=nPeriods)
#Matrix of coefficients
A=matrix(data=c(0.1,0.1,0.1,0.1),nrow=2,ncol=2)
#Auxiliary matrix of coefficients
B=matrix(data=c(0,0,0,0),nrow=2,ncol=2)
#Identity matrix
I <- diag(2)
#Production vector
x=matrix(data=0,nrow=2,ncol=nPeriods)
```

```
#Fully-adjusted production vector
x_star=matrix(data=0,nrow=2,ncol=nPeriods)
#Final demand vector
d=matrix(data=0,nrow=2,ncol=nPeriods)
#Price vector
p=matrix(data=c(0.9,1.1),nrow=2,ncol=nPeriods)
#Share of government consumption
sigma=matrix(data=c(0.4,0.6),nrow=2,ncol=nPeriods)
#Share of households consumption
beta=matrix(data=c(0.6,0.4),nrow=2,ncol=nPeriods)
#General price level
pa=matrix(data=0,nrow=1,ncol=nPeriods)

#MODEL

#Define time
for (i in 2:nPeriods){

  for (iterations in 1:20){

  if (i>=15){g[1,i]=20}
        #Government expenditures passes from 0 to 20 after 15 periods

  #Total consumption
  if (i<=2) {c[,i] = alpha1*yd[,i-1] + alpha2*h_h[,i-1]}
  else{c[,i] = alpha1*yd[,i-1]/pa[,i-1] + alpha2*h_h[,i-1]/pa[,i-1]}

  #Final demand/consumption vector
  d[,i] = beta[,i]*c[,i] + sigma[,i]*g[,i]

  #Gross production vector (full adjustment)
  x_star[,i] = solve(I-A) %*% d[,i]

  #Gross product in nominal terms
  y[,i] = t(p[,i]) %*% (I+B) %*% d[,i] #t(p[,i]) %*% x[,i]

  #Net product in nominal terms
  yn[,i] = t(p[,i]) %*% d[,i]

  #Employment
  n[,i] = (I2[,i]/pr[,i]) %*% x[,i]

  #Endogenous price vector (using Hadamard product)
  p[1,]  =  (p[2,]*A[3]*(1+mu[1,]) + w/pr[1,]) /(1 - A[1]*(1+mu[1,]) )
  p[2,]  =  (p[1,]*A[2]*(1+mu[2,]) + w/pr[2,]) /(1 - A[4]*(1+mu[2,]) )

  #Endogenous mark-up
  mu[1,i] = mu10 + mu11*(x_star[1,i-1]-x[1,i-1])
  mu[2,i] = mu20 + mu21*(x_star[2,i-1]-x[2,i-1])

  #General price level (faced by households)
  pa[,i] = t(p[,i]) %*% beta[,i]

  #Tax payments
  t[,i] = theta*yn[,i]
```

```
    #Disposable income
    yd[,i] = yn[,i] - t[,i]

    #Supply of cash money
    h_s[,i] = h_s[,i-1] + t(p[,i]) %*% sigma[,i] * g[,i] - t[,i]

    #Cash held by households
    h_h[,i] = h_h[,i-1] + yd[,i] - c[,i]*pa[,i]

    }

    #OUT-OF-ITERATION CALCULATIONS

    #Gross production vector (adjustment over time)
    if(i>=15){B = B + A%^%(i-14)}
    else{B = c(0,0,0,0) }
    x[,i] = (I+B) %*% d[,i]

}

#PLOT CONSISTENCY CHECK
plot(h_h[1,2:65]-h_s[1,2:65],type="l",col=2,ylim=range(-0.1,0.1))
```