LUBS 5228M

Understanding the Global Economy:
Capitalist Institutions, Growth and Crises

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Lecture 2

Early Keynesian models of growth

The Harrod-Domar model

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Box 1. Introduction

Starting from the 1930s, Keynes and his colleagues/pupils developed a new theoretical system based on the analysis of macroeconomic aggregates. That was the beginning of the **Keynesian revolution** in macroeconomics.

The original formulation of the Keynesian theory was concerned with the determination of income and employment levels. In short, when aggregate demand is deficient (relative to full employment output), the economic equilibrium will be established at less than full employment level.

Given the propensity to consume/save, it may well happen that the investment undertaken by firms (as determined by the expected rate of profit, given the bank interest rate) is lower than the amount of saving at the full-employment level of income.

However, investment decisions not only affect the demand, but also lead to a change in the stock of capital. While Keynes took into account the short run impact of investment decisions on demand, he chose to abstract from their long run impact on **productive capacity**.

Box 2. The equilibrium condition

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Consider a closed economy with no government sector. The aggregate demand of goods can be defined as: $AD = CONS + INV$. The market equilibrium condition is:

$$\boxed{AS = AD}$$

where AS is the aggregate supply of goods. Notice that AS equals the total amount of income distributed, call it Y. The latter, in turn, is spent for consumption ($CONS$) and/or saved (SAV). Consequently:

$$\cancel{CONS} + SAV = \cancel{CONS} + INV$$

From which we obtain:

$$\boxed{SAV = INV}$$

It defines the macroeconomic equilibrium condition of the economy. Notice that $SAV_{(ex\ post)} \equiv INV$ does not entail $SAV_{(ex\ ante)} = INV$. The former is an **identity**, the latter is an **equilibrium condition**.

1. The Keynesian multiplier

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The extension of the Keynesian model to the **long run** relies on the analysis of the interaction between the Keynesian 'multiplier' and the 'accelerator'.

As mentioned in **Lecture 1** (**Appendix**), the multiplier can be easily derived from the equation of the national income, $Y = CONS + INV$. Given the consumption function ($CONS = C_0 + c \cdot Y$), the change in national income generated by a change in the investment level is:

$$\Delta Y = \frac{1}{1-c} \cdot \Delta INV$$

where INV is the aggregate investment (autonomous demand) of firms and c is the marginal propensity to consume of households ($0 \leq c < 1$).

The term $1/(1-c) \geq 1$ is named the investment **multiplier**: an increase (decrease) in the investment level (or other demand's autonomous components) entails an increase (decrease) in national income which is a multiple of the initial expenditure (Kahn 1931, Keynes 1936).

2. The accelerator

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The second pillar of early Keynesian models of growth was the accelerator mechanism (relative to the investment in fixed capital).

Let us call a the **marginal capital-output ratio**. It defines the increase in the value of fixed capital which is necessary to increase the output by a marginal amount ($a = \Delta K/\Delta Y = INV/\Delta Y$). Harrod defined it as the value of the capital goods required for the production of a unit increment of output. The **accelerator** principle states that:

$$INV = a \cdot \Delta Y^*, \quad \text{with: } a > 1$$

where ΔY^* is the expected increase in aggregate demand.

The expectation of an increase in aggregate demand leads firms (or entrepreneurs) to undertake new investments.

Notice that $a > 1$ because the growth in the value of capital (i.e. in the expenditure for the purchase of new capital goods) is usually higher than the expected demand growth.

3. The Harrod-Domar model

This model was developed by both Roy Forbes Harrod and Evsey David Domar in various works published in the 1930s-1950s:

$$SAV_t = s \cdot Y_t \qquad \textit{multiplier}$$

$$INV_t = a \cdot \Delta Y_t^* \qquad \textit{accelerator}$$

$$SAV_t = INV_t \qquad \textit{equilibrium condition}$$

where SAV_t is the aggregate saving, $s = (1 - c)$ is the marginal propensity to save, and $\Delta Y_t^* = (Y_{t+1}^* - Y_t)$ is the expected change in aggregate demand.

Assuming that the expected change equals the actual change in demand ($\Delta Y_t^* = \Delta Y_t$), we obtain the **equilibrium solution**:

$$\frac{\Delta Y_t}{Y_t} = \frac{s}{a} = G$$

where G is the **warranted rate of growth**, i.e. the rate that guarantees the dynamic equilibrium (i.e. if firms achieved it, they would maintain it).

4. Graphical representation

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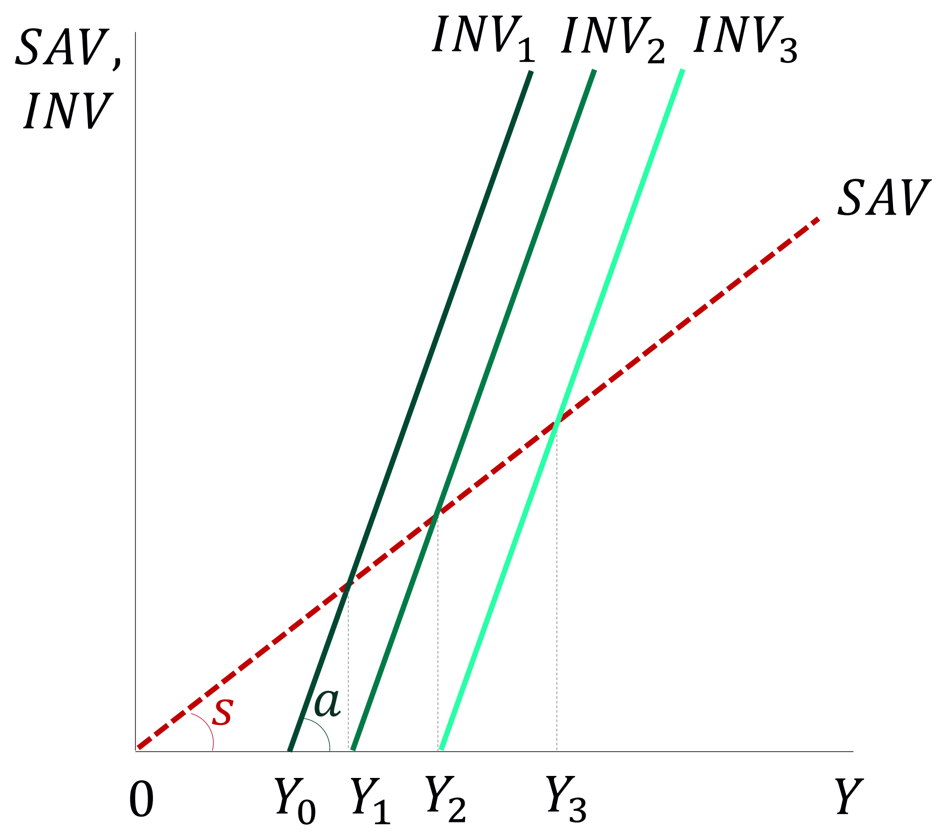
Saving-investment equilibrium in period 1 is attained when the level of income is Y_1. This level is higher than the previous one by the amount: $Y_1 - Y_0$.

The warranted rate of growth, is given by $G_1 = (Y_1 - Y_0)/Y_0$.

In period 2, the investment function shifts to INV_2 (as a is given). The new equilibrium is established where INV_2 intersects the SAV line. This occurs at the income level of Y_2.

The warranted rate of growth is: $G_2 = (Y_2 - Y_1)/Y_1 = (Y_1 - Y_0)/Y_0 = G$.

Given s and a, the warranted rate of growth remains unchanged. The same goes for the subsequent period…



5. The natural growth rate

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Two issues are apparent. First, the model is **unstable** dynamically. If the actual growth rate – call it $g_t = \Delta Y_t / Y_t$ – is higher (lower) than G, then the accelerator will increase investment more (less) than necessary.

The multiplier will increase the demand at a higher (lower) rate compared to the expected rate. This, in turn, will augment (reduce) expectations about future demand, thereby fostering the disequilibrium dynamics.

Second, the summation of the growth rates of population, n, and labour productivity (accounting for technological progress), π, gives the **natural rate of growth**: $G_n = n + \pi$.

G_n is the maximum rate of growth allowed by population dynamics, technological improvements, and availability of natural resources. It is the highest attainable growth rate, given the resources of the economy.

Does the (actual) growth rate gravitate towards the natural one?

6. Instability and sub-optimality

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No, it doesn't. Since the model is **unstable**, there is no automatic adjustment mechanism ensuring full employment and price stability in the long run.

More precisely:

- if $g_t > G_n \Rightarrow$ **inflation**, since the growth rate of demand exceeds the growth rate of production/supply
- if $g_t < G_n \Rightarrow$ **unemployment**, since aggregate demand is insufficient to absorb the available resources of the economy

Since a, n, π and s are set independently of each other, the condition defined by $g_t = G = G_n$ could hold just by chance.

~~Optimality~~

~~Stability~~

7. Simulation of a H-D model

Figure 1. Ouput (absolute value)

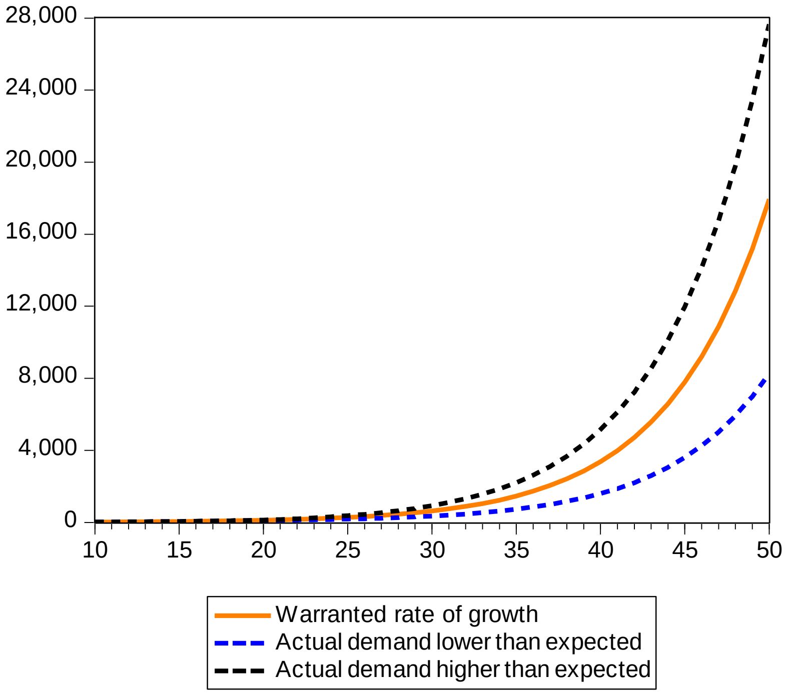
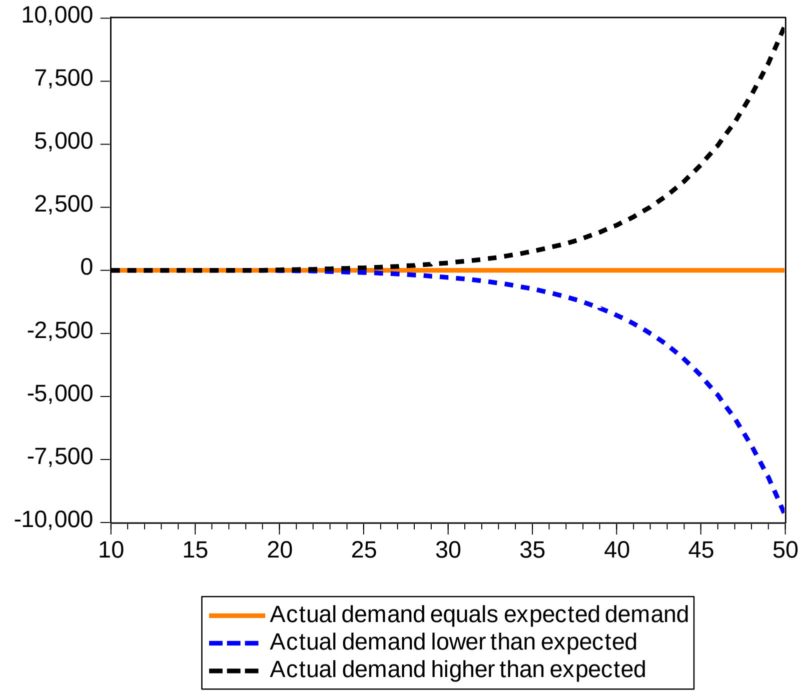


Figure 2. Output gap (absolute value)



H-D model's equations

(1) $Y_t = CONS_t + INV_t$

(2) $SAV_t = s \cdot Y_{t-1}$

(3) $INV_t = a \cdot (Y_t^* - Y_{t-1})$

(4) $CONS_t = Y_t - SAV_t$

(5) $Y_t^* = Y_t + \varepsilon$

(shock $\varepsilon = \pm\varepsilon_0$, period 20)

If we assume that $G = G_n$, then:

- Balanced growth ($g = G_n$)
- Unemployment and deflation ($g < G_n$)
- Inflation ($g > G_n$)

(Numerical simulation with EViews 6)

7. Simulation of a H-D model (cont'd)

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Figure 1b. Output gap (log)

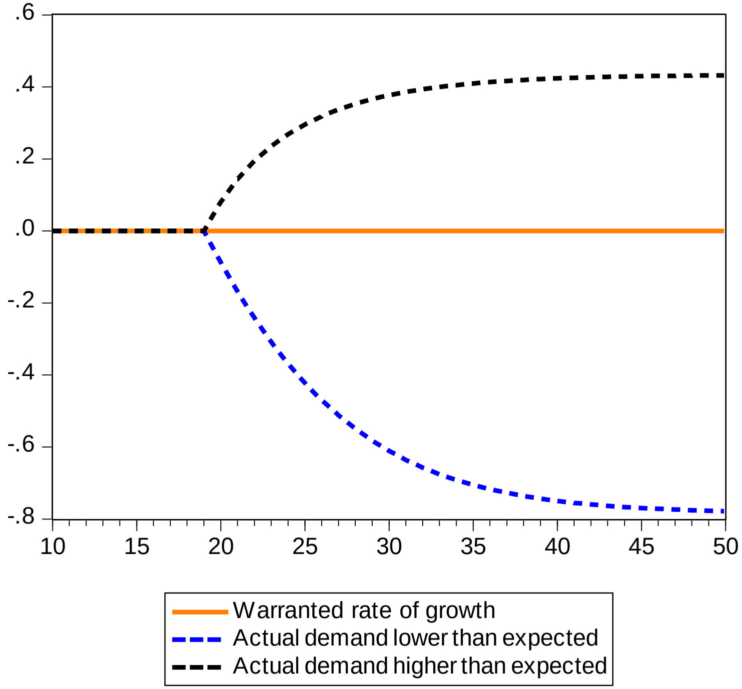
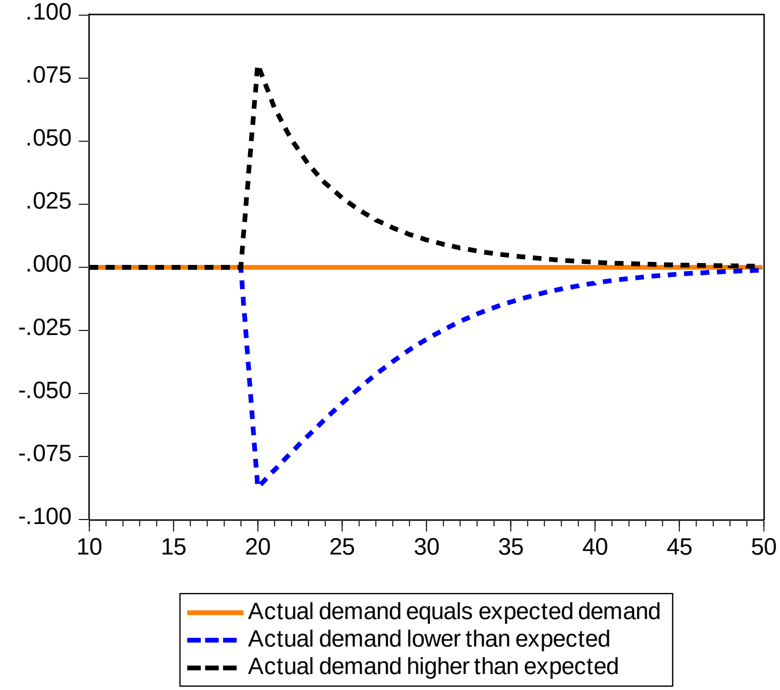


Figure 2b. Growth rate gap



Notice: we get the same findings if we run Chow (1985)'s model for Chinese economy (augmented with exp.)

(1) $Y_t = CONS_t + INV_t$

(2) $CONS_t = \gamma_0 + \gamma_1 \cdot Y_t + \gamma_2 \cdot C_{t-1}$

(3) $INV_t = a_1 \cdot (Y_t^* - Y_{t-1}) + a_2 \cdot INV_{t-1}$

(4) $SAV_t = Y_t - CONS_t$

(5) $Y_t^* = Y_t + \varepsilon$

(shock $\varepsilon = \pm\varepsilon_0$, period 20)

(Numerical simulation with EViews 6)

8. Theoretical implications

Capitalist economies are unstable systems: there is no inner market force that drives the actual rate of growth (g_t) towards the guaranteed rate of growth (G).

Any output gap, however small, is doomed to increase over time. Under a laissez-faire regime, disequilibrium is a self-feeding process.

In addition, if one assumes that the actual rate of growth matches the warranted rate of growth, nothing guarantees that the latter, in turn, equals the natural rate of growth (G_n).

The condition $g_t = G = G_n$ is called the **balanced growth equilibrium**. Joan Robinson named it (ironically) the **golden age** of capitalism.

It defines ‘a mythical state of affairs not likely to obtain in any actual economy’, because the four key variables (a, n, π and s) are determined independently of each other (they are treated as exogenous in the model).

9. Determinants of GDP growth

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The original aim of the H-D model was to analyse the business cycle. But it was later adapted to explain **economic growth** (e.g. Chow 1985, 2010).

It was used to argue that the rate of growth of an economy increases as:

- the saving and hence (?) the investment in fixed capital increase
- the marginal capital-output ratio reduces, i.e. the 'marginal productivity of capital' (?) increases

Corollary: capital is scarce in less economically developed countries (LDCs) because the rate of saving is low. That dampens economic growth.

It is necessary to increase saving rates and investment in 'highly-productive' fixed capital to boost economic development.

Problem: s depends on Y and Y depends on INV!

Notice: funding investment with foreign capitals ends up leading to political dependence, as well as economic and financial instability (ext. imbalances).

Box 3. Investment and growth in EA11

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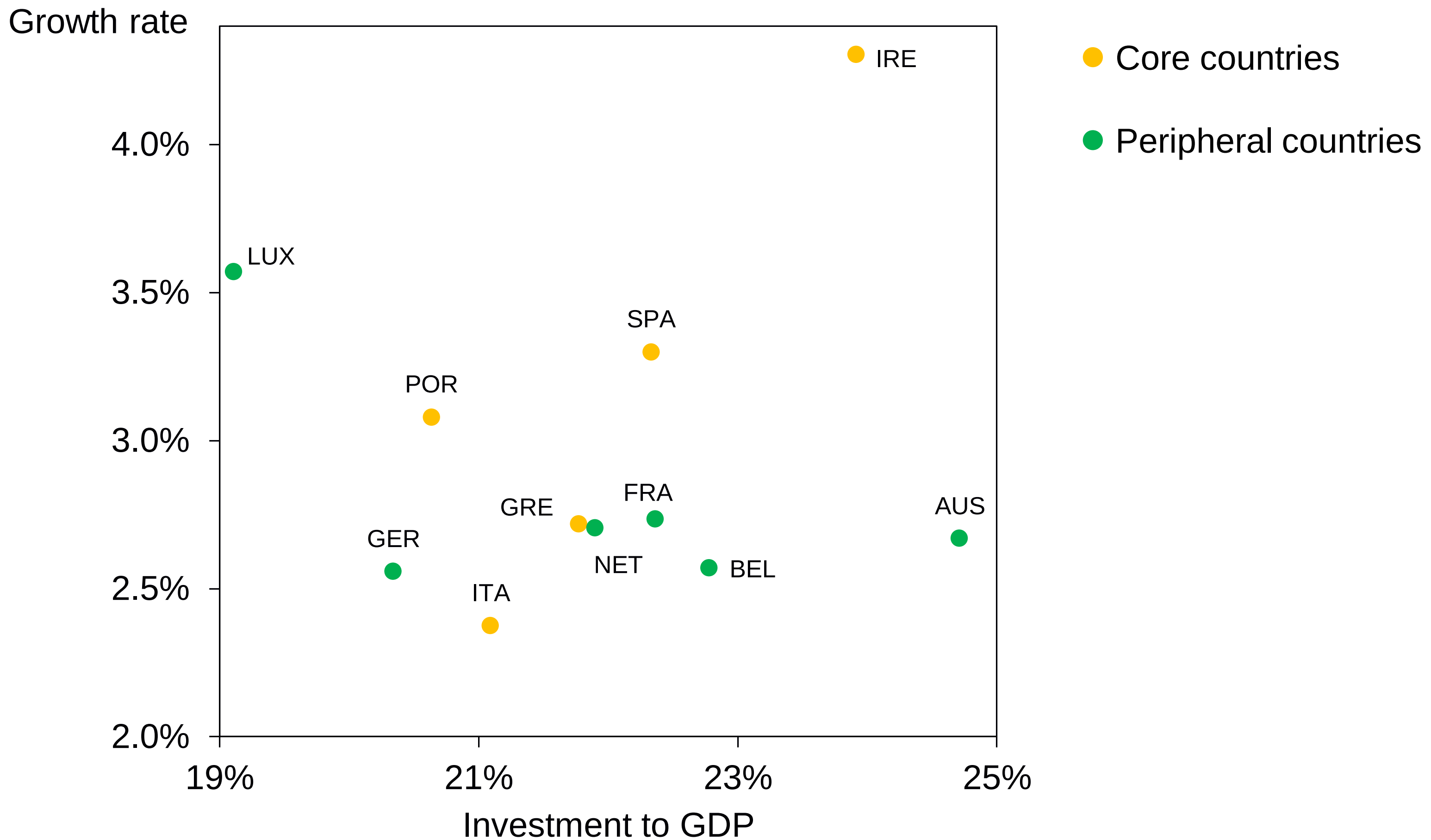

Source: AMECO 2016

Note: average values (1960-2017); GDP and investment (gross fixed capital formation) expressed at 2010 prices, mrd EURO-National Currency; growth rate = d(log(GDP))

Box 3. Investment and growth in EA11 (cont'd)

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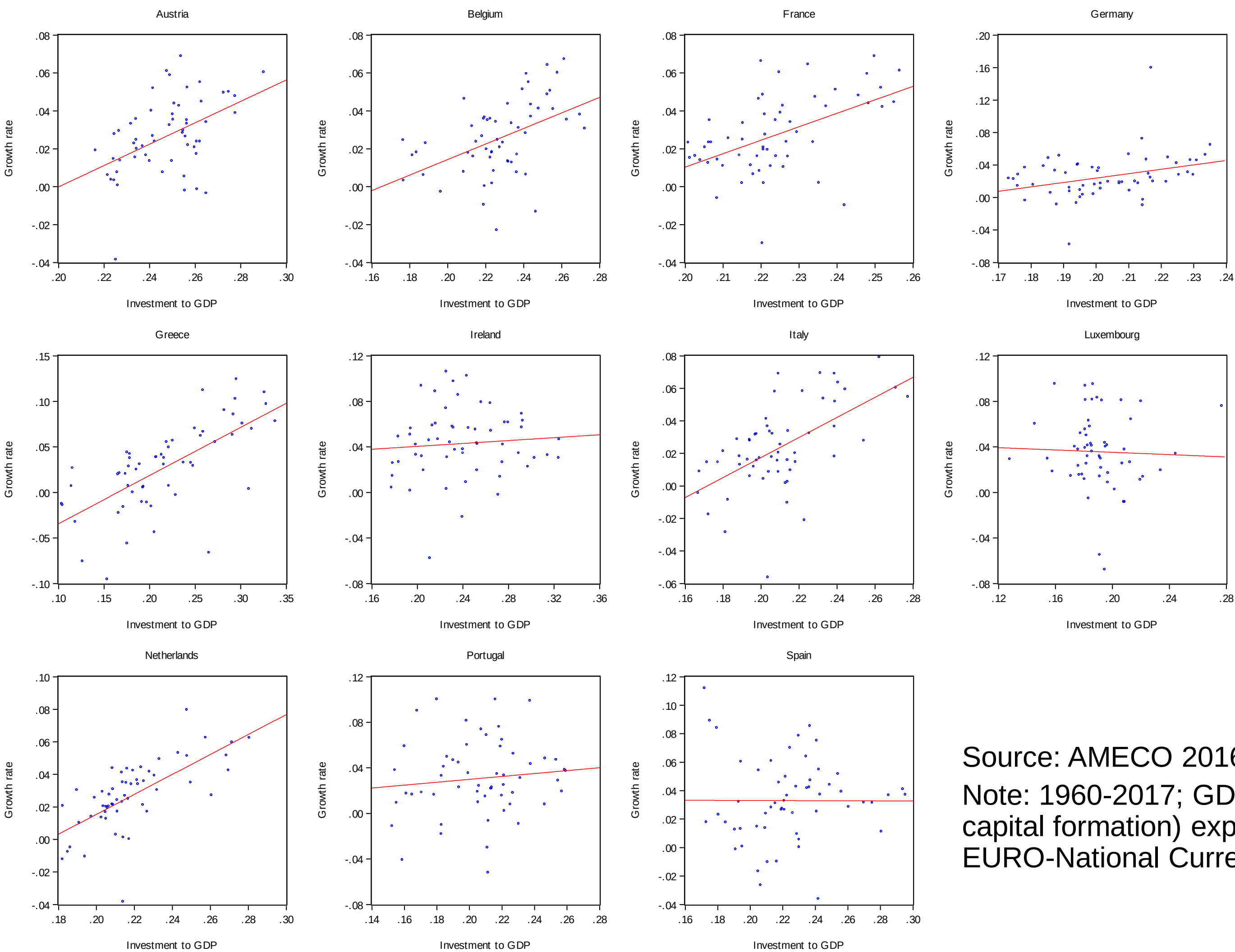

Source: AMECO 2016

Note: 1960-2017; GDP and investment (gross fixed capital formation) expressed at 2010 prices, mrd EURO-National Currency; growth rate = d(log(GDP))

10. Assumptions

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The **original H-D** model relies on the following assumptions:

- Output is a function of capital stock, i.e. $Y = f(K)$
- Constant returns to scale and constant capital-output ratio, i.e. $f(\alpha \cdot K) = \alpha \cdot Y$ and $a = \bar{a}$, so that: $\frac{dK}{dY} = \frac{K}{Y} = a = \bar{a}$
- Capital is a necessary input, i.e. $Y_{K=0} = f(0) = 0$

The **'adapted' H-D model** relies on further assumptions:

- ex-ante equilibrium, i.e. $SAV_{(ex\ ante)} = INV$
- optimal equilibrium (full employment), i.e. $N = N^f$

11. Main limitations

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When used to analyse economic growth, the H-D model shows a number of limitations:

1. The assumption of an initial **full-employment** level of income looks unrealistic for many economies (think of developing countries).

2. The model is based on **fixed capital-output (and capital-labour) ratios**, but this again sounds unrealistic for many countries (e.g. the growth of capital stock in developing countries usually does not match with the growth of labour force).

3. The capital to output ratio (a) can be rather **volatile**, because of shortages, bottlenecks, and market imperfections. The same goes for the propensity to save (s).

4. It is an aggregative model: **no room for cross-sector dynamics** (i.e. for structural and institutional changes) and hence no possible guide for **industrial policy**.

12. General policy implications

Without any government intervention mature economies would be doomed to either secular stagnation or prolonged inflation.

Both Harrod and Domar believed that **unemployment**, coupled with **chronic deflation**, was the most likely condition for advanced countries (since the propensity to save is too high compared to investment pace, i.e. effective demand is likely to fall short of productive capacity).

The State should play a stabilizing role in advanced economies and even a developmental role in developing (and ‘troubled’) countries.

Simplified though it is, the Harrod-Domar model allows defining overall **targets** for income and investment, as well as checking the consistency of those targets.

13. Final remarks

H-D-like models were developed in the inter-war period, but gained momentum in the golden age of capitalism (from the 1950s).

They provided the ground for old and new Neoclassical models of growth (see Hussein and Thirlwall 2000!). If one assumes that a can be targeted by firms (i.e. smooth substitutability of inputs), long run balanced growth and full employment can be achieved by the economy (**Lecture 3**).

However, the H-D model did not aim to show that capitalism is a balanced system. On the contrary, Harrod maintained that capitalism is characterised by persistent cyclical fluctuations and/or long run stagnation. This recalls the Marxian reproduction schemes (see **Lecture 6**).

Starting from the 1980s, the concept of ‘accelerator’ has been applied – by Ben Bernake and his colleagues – to the analysis of the effect of financial frictions on the business cycle. The ‘financial accelerator’ mechanism has become very popular (among DSGE modellers) after recent financial crises.

14. References

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CORE READING

- ✓ Screpanti, E. and Zamagni, S. (2005) An Outline of the History of Economic Thought, Oxford University Press (sections in Second Edition: 7.1.5, 7.1.6) ***online***

ADDITIONAL AND BACKGROUND READING

- ✓ Chow, G. C. (1985) A Model of Chinese National Income Determination, Journal of Political Economy, 93(4): 782-791
- ✓ Chow, G.C. (2010) Note on a model of Chinese national income determination, Economics Letters, 106(3):195-196 (pdf)
- ✓ Hussein, K. and Thirlwall, A. P. (2000) The AK Model of "New" Growth Theory Is the Harrod-Domar Growth Equation: Investment and Growth Revisited, Journal of Post Keynesian Economics, 22(3): 427-435

Next lecture (3)

Neoclassical models of growth