Reproduction, Innovation and the Profit Rate: Towards an Heterogeneous Agent-Based Approach

(*First draft: not even a working paper!*)

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Abstract

This paper aims at revisiting Marx's theory of profit in the light of recent developments in non-neoclassical economic modelling. This should help address three interconnected questions: does the original *tendency for the profit rate to fall* story hold? What is the impact of labour-saving innovations? What is the significance of the Okishio's theorem? The method used is quantitative. Comparative dynamics exercises are performed, using computer simulations, to test the reaction of an amended *enlarged reproduction* model to shocks. This method is coherent with Marx's accounting approach to the extraction, circulation and destruction of macro-monetary value and surplus-value. Besides, it is also consistent with Marx's awareness of the complex nature of social systems. The key finding is that the Okishio' theorem is no longer generalisable once the assumption of a *representative agent* is replaced with a set of *heterogenous agents* marked by different propensities to innovate.

Keywords: Marxian Economics, Stock-flow Consistent Models, Agent-based Models

JEL Classification: B51, B52, E11

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1 Introduction

The view of the economic system as a circular flow of payments and revenues was pioneered by François Quesnay in his *Tableau Économique* (1758). Quesnay likened the physiology of a stylised agricultural economy to the blood flow in the human body. One century later, Marx recovered and developed Quesnay's insights in his 'reproduction schemes' (RS). The latter define the preconditions allowing a capitalist economy to reproduce over time (Marx 1885,[6] chapters 20 and 21). More precisely, equilibrium conditions are defined in terms of interdependences between industries, that is, in terms of the flows of goods that must be supplied by each industry to meet exactly other industries' demands for inputs. Notice that the RS do not aim to prove that capitalist economies can always meet equilibrium conditions. On the contrary, they allow Marx to argue that disequilibrium or sub-optimal equilibria are the normal state. For there is no inner mechanism that makes capitalist firms' decisions mutually consistent.

While Marx never provided a complete mathematical model of the (enlarged) RS, he left several notes and numerical examples, which can be expressed as systems of difference (or differential) equations. In fact, there is a well-established tradition of dynamic modelling carried out by Marxist economists throughout the 1970s, who have been inspired by the Marxian RS (e.g. Harris 1972[5], Bronfenbrenner 1973[1], Morishima 1973[8]). It undeniable that RS models have lost momentum ever since. However, some exceptions can be identified in the last few years, notably, Olsen 2015[10], Cockshott 2016[2], and Veronese Passarella 2019[11].

This paper aims at revisiting Marx's theory or profit in the light of recent developments in non-neoclassical economic modelling. This should help address three interconnected questions: *a*) does the original *tendency for the profit rate to fall* (TPRF) story hold? *b*) What is the impact of labour-saving innovations? *c*) What is the significance of the Okishio's theorem? The method used is quantitative. First, an amended *enlarged reproduction* model is developed, building upon Marx's numerical examples. Second, comparative dynamics exercises are performed, using computer simulations, to test the reaction of model's endogenous variables to shocks. Stock-flow consistent and agent-based modeling techniques are employed. This is coherent with Marx's accounting approach to the extraction, circulation and destruction of monetary value and surplus-value. Besides, it is consistent with Marx's awareness of the complex nature of social systems.

The next sections are organised as follows. Section 2 provides a general recap of the benchmark model of enlarged reproduction. Model's assumptions and key features are presented and discussed in Section 3. In Section 4 a preliminary experiment is conducted by testing key variables' reactions to a negative shock to the average retention rate on I-firms' profit. The model is then used to discuss the impact on profitability of labour-saving innovations undertaken by I-capitalists. Section 5 offers some concluding remarks.

2 The RS Model: A Recap

The enlarged RS model depicts a growing capitalist economy that moves forward in time, t, and is made up of two sectors or departments: a sector producing capital or investment goods (called 'department I' by Marx), defined by the subscript 'I'; and a sector producing consumption goods (named 'department II'), defined by the subscript 'C'. For the sake of simplicity, it is assumed that each production process takes exactly one period to be completed (that is, the intra-period turnover rate is unity for both the C-sector and the I-sector). Commodities are produced by means of capital goods and labour inputs. Labour force is plentiful (*reserve army of labour* assumption). The availability of workers does not form a binding constraint on the level of employment. A net product arises both in real and monetary terms in each sector, where it is distributed as wages to the workers and surplus value (or profit) to the capitalists. Variables are expressed in monetary terms (current

prices) when not otherwise specified.

As is well known, Marx distinguishes the *variable* component of capital from its *constant* component. The former roughly matches the wage bill paid by the industrial capitalists to the workers in exchange for their labour power. It covers the part of the total working day that is devoted to the production of *subsistence* for workers.[1] The latter defines the value of capital inputs (that is, fixed and circulating capital net of wages) accumulated in the each department. Therefore, the change in each sectoral variable capital is:

$$V_j - V_{j,-1} = \frac{S_{j,-1} \cdot \theta_j}{1 + q_j} \tag{1}$$

where $j = C, I$ identifies the sector, θ_j is the retention rate on profits, and q_j is the so-called 'organic composition of capital' (OCC). The OCC is the ratio between the constant capital and the variable capital necessary that are required to start the production process. It is taken as an exogenous variable of the model.

The value of constant capital invested in each sector is:

$$C_j = V_j \cdot q_j \tag{2}$$

The mass of surplus-value created in each production process is:

$$S_j = \epsilon_j \cdot V_{j,-1} \tag{3}$$

where ϵ_j is the exploitation rate, which is also taken as an exogenous variable of the model (as it depends on a variety of social, institutional and political factors).

If the competition between different sectors is assumed away, the non-contestable sectoral profit rate is:

$$r_j = \frac{S_j}{C_j + V_j} \tag{4}$$

The rate of accumulation in each sector is:

$$g_j = \frac{\frac{\theta_j \cdot S_j}{1+q_j}}{V_j} = \epsilon_j \cdot \theta_j \cdot \frac{1}{1 + q_j} \tag{5}$$

Particularly, the rate of accumulation of constant capital in the I-sector is:

$$g_I = \epsilon_I \cdot \theta_I \cdot \frac{1}{1 + q_I} \tag{6}$$

The accumulation of constant capital in the C-sector is:

$$S_C \cdot \theta_C \cdot \frac{q_C}{1 + q_C} + C_C = Y_I - C_I - S_I \cdot \theta_I \cdot \frac{q_I}{1 + q_I} \tag{7}$$

where Y_I is the I-sector output value. It is implicitly assumed that the value realised on the market (circulation) matches the value created *in potentia* in the production sphere. In other words, demand deficiencies are a assumed away.

The accumulation of variable capital in the C-sector is:

$$S_C \cdot \theta_C \cdot \frac{1}{1 + q_C} + C_C = \left(Y_I - C_I - S_I \cdot \theta_I \cdot \frac{q_I}{1 + q_I} - C_C\right) \cdot \frac{1}{q_C} \tag{8}$$

[1] For the sake of simplicity, we put aside the so-called 'transformation problem' by assuming that the monetary value added of the economy expresses the amount direct labour that is abstractly and socially necessary to produce throughout a certain period. In other words, a *simultaneous* and *single-system* interpretation of the Marxian labour theory of value is used, in the wake of Duménil and Foley (2008)[3]. Coherently, the variable capital should be better defined as the *unallocated purchasing power* of workers, meaning the quantity of direct labour expressed by the commodities that the workers can buy on the market in exchange for their money wages.

Consequently, the equilibrium rate of growth of the C-sector is:

$$g_C = \frac{S_C \cdot \theta_C \cdot \frac{q_C}{1+q_C}}{C_C} = \frac{Y_I - C_I - S_I \cdot \theta_I \cdot \frac{q_I}{1+q_I}}{C_C} - 1 \tag{9}$$

This condition assures the consistency of C-sector capitalists' investment decisions with I-sector capitalists' production and accumulation plans. Therefore, it guarantees the gravitation of the economy towards its (enlarged) reproduction equilibrium. However, this ideal state is extremely unlikely to be matched and maintained in practice. In fact, Marx uses the RS equilibrium condition to argue that real-world capitalist economies are always in a state of disequilibrium or trapped into sub-optimal equilibria.

The economy-wide balanced growth rate is the uniform rate that prevails when C-capitalists fully adjust their production plans to fit I-capitalists' autonomous decisions:

$$g = g_C = g_I = \epsilon_I \cdot \theta_I \cdot \frac{1}{1+q_I} = \theta_I \cdot r_I \tag{10}$$

Using $g_C = \epsilon_C \cdot \theta_C/(1+q_I)$, we obtain the *enlarged reproduction equilibrium condition*:

$$\frac{\theta_C}{\theta_I} = \frac{\epsilon_I}{\epsilon_C} \cdot \frac{1+q_C}{1+q_I} \tag{11}$$

The ratio of sectoral retention rates must be proportional to sectoral organic compositions of capital, given the exploitation rates. Since these variables are independent of each other, nothing ensures that condition (10) is met in practice.[2]

In principle, balanced growth is possible, as the expansion of production in one sector enlarges the market for the other. However, 'The rate of growth of production in the various branches of production is determined [also] by the *uneven development* of the conditions of production, rather than by the different rates of growth of the markets for their products' (Clarke 1990). This leads to a disproportional development of the two sectors, which is the form taken by the inner tendency of capitalism to over-accumulation and crisis. By constrast, enlarged reproduction conditions are matched if sectors grow all at the same pace. This bears resemblance to the *Cambridge distributive equation*, $r = g/\theta$, interpreted as a dynamic investment function in a two-sector economy. While the (average) I-sector retention rate is an exogenous, the (average) C-sector retention rate must behave like a *buffer* to ensure the equilibrium:

$$\theta_C = \frac{g_C \cdot (1+q_C)}{\epsilon_C} \tag{12}$$

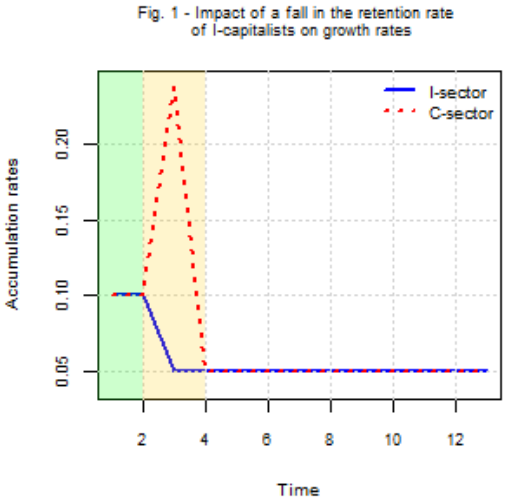
Historically, this *stabilising* role has been preformed by the State (the 'Big Government' and the 'Big Bank', as Hyman Minsky would call it) and the foreign sector (imperialism, neo-mercantilism). Fig. 1 and Fig. 2 show what happens when the retention rate on profit of the I-sector reduces. In order for the equilibrium to be restored, the retention rate of the buffer sector (here the C-sector) must reduce in the medium run, thus equalising the accumulation rates across the two sectors. In fact, the fall in the C-sector retention rate outstrips the initial fall in the I-sector retention rate.

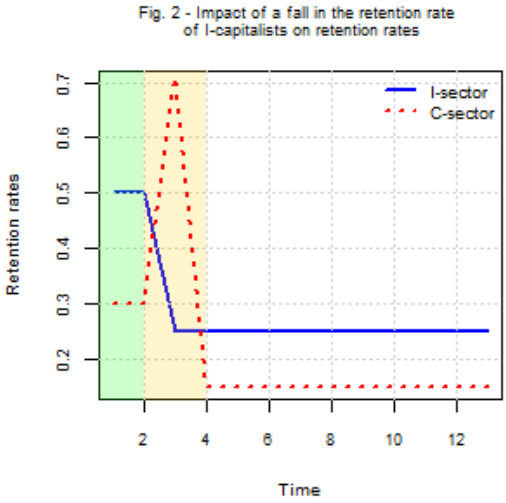
3 The Amended Model

3.1 Model assumptions

The benchmark RS model presented in Section 2 provides a simple but insightful explanation of instability and crisis tendencies that characterise unregulated capitalist economies. On

[2] For a complete overview of the RS model, including the role of sectoral turnover rates, we refer to Veronese Passarella (2019)[11].





the one hand, Marx's grim predictions about the future of capitalism fit well with the history of early-industrialised countries from the end of the Victorian Era to the Second World War. On the other hand, the RS model provides a theoretical rationale for the stabilising function performed by the government sector in major western economies since the 1930s.

However, three aspects are not covered by the benchmark RS model:

1) The role of commercial banks and financial intermediaries, which have been taken center stage in the last decades (financialisation).

2) The impact of cross-sector investments and the alleged convergence of sectoral profit rates due to competition forces (profit rate equalisation).

3) The role of heterogeneity and interaction among economic agents (horizontal and vertical class struggle).

This paper focus on, and aims at bridging, the third gap, particularly the horizontal conflict within the capitalist class. The fact is that the benchmark RS model lacks the *granularity* that would be necessary to analyse the interaction of agents between and within social classes, thus identifying capitalism's laws of motion as emerging behaviours of a complex system.

The main feature of the model presented in this section are listed below:

- Workers' saving, capital depreciation and the government sector are assumed away.
- Money is implicitly regarded as an endogenous flow (created by the banking sector) that adjusts to firms' demand for loans.
- There is no rent, while the bank interest rate is null.
- C-sector capitalists play a passive role (acting as the buffer sector), as they have to adjust their production plans to I-sector capitalists' decisions.

Initial values of variables and parameters are calibrated using Marx's own examples and the available literature. Besides, it is also assumed that:

- There are 100 capitalists in the I-sector and 100 capitalists in the C-sector, which can record losses but never go bankrupt. The reserve army of labour is infinite.

- I-sector capitalists are split into *innovators*, who are eager to introduce labour-saving techniques of production to boost their (short-run) profit rate, and routine capitalists or *latecomers*, who must catch up with new technologies and methods.

- Capital accumulation and innovations are the key driving forces of the system and so is the horizontal conflict within the capitalist class. The vertical class struggle between the workers and the capitalists is assumed away instead.

Table 1: Initial conditions

Description	Parameter values
Total number of capitalists	$N = 200$
Number of capitalists in I-sector	$N_I = 100$
Starting percentage of innovators in I-sector	$\rho_0 = 0.05$
Number of capitalists in C-sector	$N - N_I = 100$
Number of multiple simulations	$MC = 200$
Average exploitation rate in I-sector	$\epsilon_I = 1$
Average exploitation rate in C-sector	$\epsilon_C = 1$
Retention rate in I-sector	$\theta_I = 0.5$
Retention rate in C-sector	$\theta_C = 0.3$
Organic composition of capital in I-sector	$q_I = 4$
Organic composition of I-innovators	$q_I^{inn} = q_I = 4$
Organic composition of capital in C-sector	$q_C = 4\,[2]$
Organic composition of capital in C-sector after innovation	$q_C^{inn} = q_C = 4\,[2]$
Min. % of innovative to routine variable capital in I-sector	$\pi_v = 0.5$
Partner choice parameter (intensity)	$\lambda = 0.1$
Speed of adjustment of individual supply to demand in I-sector	$\sigma_I = 0.5$
Speed of adjustment of individual supply to demand in C-sector	$\sigma_C = 0.5$
Coefficient of error (innovation) function	$\rho_0 = 0.05$
Coefficient of error (innovation) function	$\rho_1 = 69$

Notice that the method used (comparative dynamics exercises through computer simulations) is inspired by the stock-flow consistent and interacting agent-based modelling literature. Experiments are conducted throughout 250 periods and using 200 Monte Carlo simulations or scenarios. Random components are added to some model coefficients (namely, I-sector individual outputs, I-sector constant capital values and exploitation rates) to mimic workers' idiosyncratic reactions and allow for a rough sensitivity test on I-sector coefficients.

While the new model is still based on the distinction between a C-sector and a I-sector, they now comprise a plurality of capitalist firms. The relationship between C-sector capitalists and I-sector capitalists is ruled by a stochastic matching mechanism: each C-capitalist randomly selects a partner from the I-capitalist group. After that, C-capitalists adjust their own production plans correspondingly. In each period prices are set in such a way to clear the market. However, real production adjusts to demand conditions in the long run. This assumption is in line with Marx's description of the reproduction process, which turns the standard neoclassical story (advocating a quantity adjustment in the short run vs. price adjustment in the long run) upside down.[3]

Innovation spread is defined as the percentage, ρ, of I-capitalists who get aware of and use the new technique of production:

$$\rho = \rho_0 + ERF\Big(\frac{t - t_0}{\rho_1}\Big) \tag{13}$$

where $0 < \rho_0 < 1$, $\rho_1 > 0$, $ERF(\cdot)$ is the error function and t_0 is the shock period.

[3] This seems to imply that firms' plants always operate close to full capacity utilisation, which is coherent with the assumption of an unregulated, fully competitive, market economy (provided that demand does never lag behind).

3.2 Initial conditions and the sequence of events

The model has been implemented using *R*.[4] Its algorithmic structure is defined by the initial conditions and the sequence of events. The initial values for parameters and variables are shown by Table 1. In each sector, the exploitation or suplus-labour rates are defined as exogenous (but not strictly deterministic) variables:

$$\epsilon_{ij} = \epsilon_0^j \cdot (1 + \zeta_{ij}) \tag{14}$$

where $i = 1, 2, ..., N_j$ identifies the individual capitalist (or firm), $j = C, I$ identifies the sector, and $\zeta_{ij} \sim U(-0.1, 0.1)$ is a random variable.

The sequence of events that take place for every period of time is as follows:

1) Each C-capitalist randomly selects a potential partner from the set of I-firms.

2) Each I-capitalist can either innovate (opting for a labour-saving technique of production) or stick to the existing technique of production.

3) If the *old* technique of production is used then:

   3.1) The variable capital invested by each individual I-capitalist is: $V_{iI} = V_{iI[-1]} + (\epsilon_{iI} \cdot V_{iI[-1]} \cdot \theta_{iI})/(1 + q_{iI})$, where the subscript i identities the i-th agent.

   3.2) The individual surplus value is: $S_{iI} = \epsilon_{iI} \cdot V_{iI}$.

   3.3) The individual constant capital is: $C_{iI} = V_{iI} \cdot q_{iI}$.

   3.4) The growth rate or capital accumulation rate is: $g_{iI} = \theta_{iI} \cdot \epsilon_{iI}/(1 + q_{iI})$.

4) If a *new*, labour-saving, technique of production is introduced then:

   4.1) The individual constant capital is identical except for stochastic shocks, which capture the uncertainty linked with the new method or machine: $C_{iI}^{inn} = C_{iI} \cdot (1 + \zeta_{ij})$.

   4.2) The individual variable capital reduces (on average), because of the higher organic composition of capital: $V_{iI}^{inn} = C_{iI}^{inn}/q_{iI}^{inn} < V_{iI}$.

   4.3) The individual surplus value that can be extracted from the workers in the production sphere is also lower, that is: $S_{iI}^{inn} = \epsilon_{iI} \cdot V_{iI}^{inn} < S_{iI}$.

   4.4) The growth rate or capital accumulation rate is: $g_{iI} = \theta_{iI} \cdot \epsilon_{iI}/(1 + q_{iI}^{inn})$.

5) Each C-capitalist makes its production plans based on their I-sector partner's decisions. If no innovations are introduced then:

   5.1) The variable capital invested by each individual C-capitalist is: $V_{iC} = V_{iC[-1]} + (e_{iC} \cdot V_{iC[-1]} \cdot \theta_{iC})/(1 + q_{iC})$.

   5.2) The individual surplus value is: $S_{iC} = \epsilon_{iC} \cdot V_{iC}$.

   5.3) The individual constant capital is: $C_{iC} = V_{iC} \cdot q_{iC}$.

   5.4) The individual output value is: $\Lambda_{iC} = C_{iC} + V_{iC} + S_{iC}$.

   5.5) The individual profit rate is: $r_{iC} = S_{iC}/(C_{iC} + V_{iC})$.

   5.6) The growth rate or capital accumulation rate is: $g_{iC} = (V_{pI}/C_{iC}) \cdot \Big[1 + \epsilon_{pI} \cdot \big(1 - \theta_{pI} \cdot \frac{q_{pI}}{(1+q_{pI})}\big)\Big] - 1$,

   where p in subscript pI marks the I-sector partner (provider) chosen by the C-sector firm (buyer).

6) If innovations are introduced in the I-sector then:

[4] The complete codes of both the aggregate model and the agent-based model are available upon request.

6.2) The organic composition of each C-firm capital does not change: $q_{iC}^{inn} = q_{iC}$.

6.2) The variable capital becomes: $V_{iC}^{inn} = V_{iC[-1]}^{inn} + (\epsilon_{iC} \cdot V_{iC[-1]}^{inn} \cdot \theta_{iC}^{inn})/(1 + q_{iC}^{inn})$.

6.3) The individual surplus value is: $S_{iC}^{inn} = \epsilon_{iC} \cdot V_{iC}^{inn}$.

6.4) The individual constant capital is: $C_{iC}^{inn} = V_{iC}^{inn} \cdot q_{iC}^{inn}$.

6.5) The individual profit rate is: $r_{iC}^{inn} = S_{iC}^{inn}/(C_{iC}^{inn} + V_{iC}^{inn})$.

6.6) The retention rate is: $\theta_{iC}^{inn} = g_{iC}^{inn} \cdot (1 + q_{iC}^{inn})/\epsilon_{iC}$.

6.7) The growth rate or capital accumulation rate is: $g_{iC}^{inn} = (V_{pI}^{inn}/C_{iC}^{inn}) \cdot \Big[1 + e_{pI} \cdot \big(1 - \theta_{pI}^{inn} \cdot \frac{q_{pI}^{inn}}{(1+q_{pI}^{inn})}\big)\Big] - 1$.

7) Individual market prices and supplies are then defined:

7.1) The individual nominal demand for I-goods faced by the capitalist i is: $D_{iI} = C_{iI} + C_{pC} + \theta_{pC} \cdot S_{pC}$.

Notice that p in subscript pC now marks the C-sector firm (buyer) associated with each I-sector firm (provider).

7.2) The individual nominal demand for C-goods faced by the capitalist i is: $D_{iC} = V_{pI} + V_{iC} + (1 - \theta_{iC}) \cdot S_{iC} + (1 - \theta_{pI}) \cdot S_{pI}$.

7.3) The real planned supply of I-goods by the i-th I-firm is: $X_{iI} = X_{iI,-1} + (\sigma_I + \zeta_{iI}) \cdot (D_{iI} - X_{iI,-1})$,
where σ_I is a parameter defining the speed of convergence of each I-firm's supply to the actual demand for I-goods. A random component is included to mimic the uncertainty that characterises expectations about the actual demand level.

7.4) If there is no innovation, the market price realised by the i-th I-capitalist is: $p_{iI}^{m} = (C_{iI} + C_{pC} + \theta_{iI} \cdot S_{iI} + \theta_{pC} \cdot S_{pC})/X_{iI} = D_{iI}/X_{iI}$.

7.5) If a labour-saving innovation is introduced, the market price realised by the i-th I-capitalist becomes: $p_{iI}^{m,inn} = (C_{iI} + C_{pC} + \theta_{iI} \cdot S_{iI}^{inn} + \theta_{pC} \cdot S_{pC})/X_{iI}$.

7.6) The standard market price realised by the i-th C-capitalist is: $p_{C}^{m} = D_{iC}/X_{iC}$.

7.7) When innovations are introduced in the I-sector, the market price realised by the i-th C-capitalist becomes: $p_{C}^{m,inn} = \big[V_{pI}^{inn} + V_{iC} + (1 - \theta_{iC}) \cdot S_{iC} + (1 - \theta_{pI}) \cdot S_{pI}^{inn}\big]/X_{iC}$.

8) For the sake of simplicity, it is assumed that the monetary expression of labour time is unity. Therefore, labour productivities and labour quantities related to each capitalist firm can be defined as follows:

8.1) The direct labour spent in each routine I-firm production process is: $L_{iI} = V_{iI} + S_{iI}$.

8.2) The direct labour spent in each innovative I-firm production process is: $L_{iI}^{inn} = V_{iI}^{inn} + S_{iI}^{inn}$.

8.3) The direct labour spent in each C-firm production process is: $L_{iC} = V_{iC} + S_{iC}$.

8.4) When innovations are introduced in the I-sector, the magnitude above becomes: $L_{iC}^{inn} = V_{iC}^{inn} + S_{iC}^{inn}$.

8.5) Labour productivity of the i-th firm of the I-sector is: $a_{iI} = X_{iI}/L_{iI}$ with no innovation, and $a_{iI}^{inn} = X_{iI}^{inn}/L_{iI}^{inn}$ if a labour-saving innovation is introduced.

8.6) Similarly, labour productivity of the i-th firm of the C-sector will be: $a_{iC} = X_{iC}/L_{iC}$ and $a_{iC}^{inn} = X_{iC}^{inn}/L_{iC}^{inn}$, respectively.

9) The spread of innovations throughout the I-sector is ruled by equation (12), which defines the percentage of innovators. It can also be regarded as the *contagion mechanism* of the model.

Equations above define individual capitalist-based variables. Sums, average values and standard deviations are then calculated at the end of each period. This is done first across individual capitalists (or firms) and then across Monte Carlo scenarios. Therefore, the general profit rate of the economy in each period t and for each Monte Carlo scenario mc is:

$$r^{[t,mc]} = \frac{S_I^{[t,mc]} + S_C^{[t,mc]}}{C_I^{[t,mc]} + V_I^{[t,mc]} + C_C^{[t,mc]} + V_C^{[t,mc]}} \tag{15}$$

where $S_I = \sum S_{iI} + \sum S_{iI}^{inn}$, $S_C = \sum S_{iC} + \sum S_{iC}^{inn}$, $C_I = \sum C_{iI} + \sum C_{iI}^{inn}$, $V_I = \sum V_{iI} + \sum V_{iI}^{inn}$, $C_C = \sum C_{iC} + \sum C_{iC}^{inn}$ and $V_C = \sum V_{iC} + \sum V_{iC}^{inn}$ (in which, as usual, the subscript i marks the i-th capitalist).

4 Findings and discussion

The following two experiments are made:

- A fall in the average retention rate on profit of capitalist firms of the I-sector.
- The launch of a labour-saving technology in the I-sector. This brings about an increase in the average organic composition of the innovators in the I-sector.

4.1 Adjustments through changes in the retention rate

Model's reaction to a fall in the average retention rate on profit of I-sector capitalists is first tested. This is a quite standard experiment in the Marxist economics literature. It is here run for calibration and model dynamics checking purposes. Model's assumptions are as follows:

- Exploitation rates across the two sectors are identical (and unity) on average. However, sectoral organic compositions of capital are different (4 for the I-sector and 2 for the C-sector). These assumptions are in line with Marx's numerical examples.
- There is a tendency for the profit rate to equalise *within* the sectors but *not across* sectors ($r_C \neq r_I$). In other words, markets are *non-contestable*.[5]

Given the assumptions above:

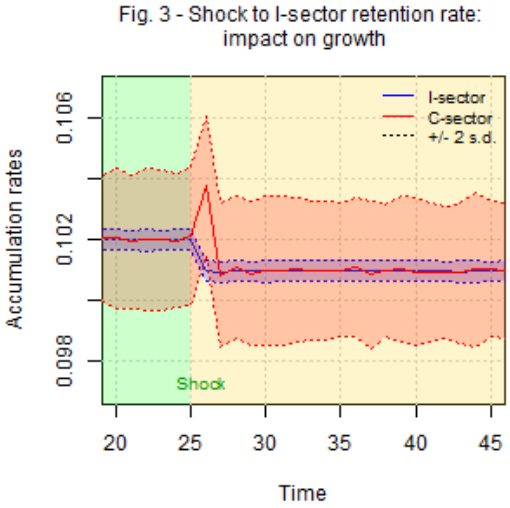
- I-sector capitalists reduce their (average) retention rate on profits.
- This is expected to lead C-sector capitalists to adjust their production plans to meet the new demand for consumer goods (in fact, this is the implication to be checked).

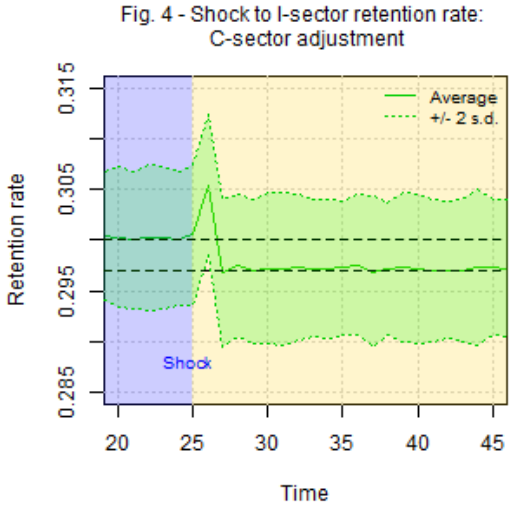
As expected, a fall in the average retention rate of I-capitalists is associated with a reduction in the I-sector accumulation rate. The average C-sector accumulation rate initially increases, due to the lower saving rate that entails a higher consumption (see Figure 3, where the shock is run in period 2). However, the C-sector accumulation rate reduces to match the I-sector rate in the medium run, due to the fall in production. Like it happens in the aggregate model presented in Section 2, the key variable is the (average) retention rate of the C-sector (Figure 4), which adjusts to fit I-capitalists' new production plans.

4.2 A dynamic confutation of the Okishio theorem?

The second experiment allows assessing the impact of agents' heterogeneity on profit rates. More precisely, I-sector capitalists are ideally split into two subgroups:

[5] Notice that, while Marx presents the profit rate equalisation tendency in Chapter 10 of the Third Volume of *Capital*, he puts that tendency aside in other chapters of the same volume.





- The *innovators* are those who are eager and ready to introduce new, labour-saving, methods or technologies in the I-sector. For the sake of simplicity, we assume that the innovation enables its users to reduce the labour costs (the variable capital) for each unit of capital invested in machinery and row materials (constant capital), without reducing the supply of I-goods. In other words, the innovation increases both the organic composition of capital and labour productivity.

- The *late-comers* (or routine capitalists) are both those who decide to stick to routine methods, technologies and machines as long as they can and those who cannot readily change their plans (because of institutional, political, technological and/or financial constraints).

Obviously, the new method or technology gradually becomes the *new normal* as the innovation spreads. A never-ending process of innovation is necessary to keep the whole capitalist class from turning into a routine sector. Under a free-market regime, this is assured by the competition between the capitalists.

In addition to the innovator vs. routine agent distinction, the following assumptions are made:

- Capitalist firms share the same initial organic composition of capital on average (that is, $E(q_{iI}) = E(q_{iC}) = 4$) across the two sector.

- As mentioned in Sections 2 and 3, innovations spread over time and across I-sector capitalists following a *smoothed* error function (see Figure A7 in the *Appendix*).

- The innovation enables its users to produce the same quantity of goods (individual supply) of the competitors by employing the same amount of constant capital but less variable capital. In principle, this can either imply that the innovation makes the labour-force more productive (because it increases the net output per labour unit) or it makes it cheaper (because less qualified workers can be employed). The first option is the one followed here.

- In order to focus on the effect of an increase in the organic composition of capital, the counteracting factors to the falling rate of profit are all assumed away.

- For the same reason, supply-side constraints and aggregate demand deficiencies are neglected. Unit prices fully adjust to clear the market in the short run (while the long run adjustment is quantitative – see Section 2).

Building on the assumptions above, two different sub-scenarios are considered:

A) The innovation turns entirely into higher real wages for the employed workers (and higher profits for innovative firms in the short run), because of the reduction in the unit price of C-goods. As a result, the real wage is unchanged for the working class (if the unemployed are included) and so is the class rate of exploitation.

B) The innovation does not fully benefit the employed workers, whose real wage rate does not growth in line with labour productivity. As a result, the real wage for the whole working class (including the unemployed) is lower, while the class exploitation rate is higher.

The former is associated with a *class-based* definition of subsistence and entails a *constant wage share*, whereas the latter is associated with an *individual-based* definition of subsistence and entails a *falling wage share* following a labour-saving innovation.

Fig. 5 - Phases of innovators' profit

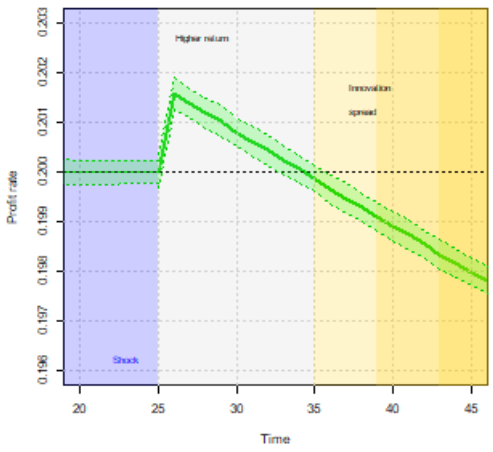


Fig. 6 - Profit rates following innovation in I-sector

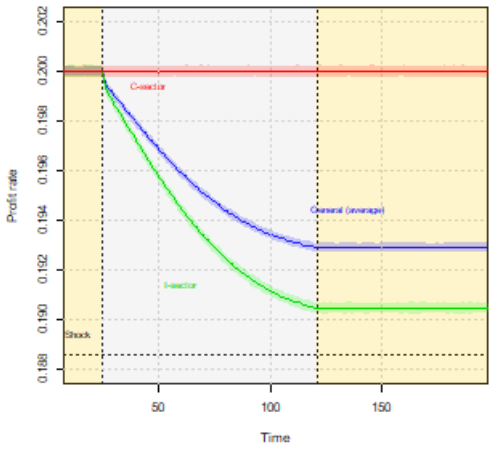
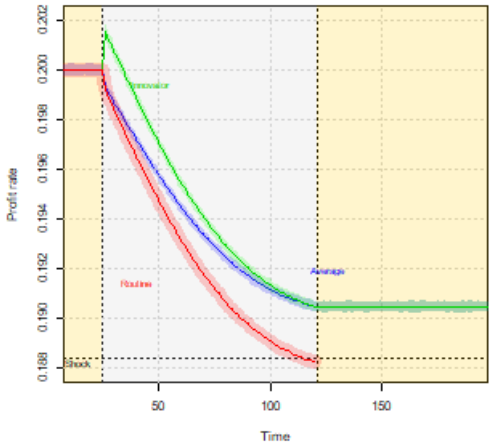


Fig. 7 - Profit rates in I-sector after innovation



A) *Class-based definition of subsistence (or constant wage share).* The impact on sectoral and individual profit rates is first tested by assuming that the employed workers fully capture the innovation-led increase in labour productivity, via a fall in the average unit price of consumption goods. The innovation is introduced starting from period 25. Figure 5 shows that the average profit rate of the innovators increases following the shock, despite the general increase in real wage rates. It remains above the pre-shock value for approximately 10 periods and falls below its initial value afterwords. The profit rate for the whole I-sector records a different trend. Figure 6 shows that it declines over time. It reaches a plateau when all the I-sector capitalists have moved to the new technology or method. This is shown by Figure 7, which displays the different trends in average profit rates for the innovators (green line), the late-comers (red line) and the whole sector (blue line), respectively. Notice that, since it has been assumed that the markets are non-contestable, the C-sector average profit rate is not affected. However, the general rate of profit is, due to the change in the average I-sector profit rate – see again Figure 6.

To sum up, the experiment shows that there can be individual incentive to innovate even if this affects the general profit rate. For innovation assures a higher individual profit rate (due to lower labour costs) in the short run. Therefore,

competition forces will push the capitalists to take their chances. However, the increase in the profit rate (associated with a one-shot innovation) is only temporary. Innovators' extra-profits decline as innovation spread reduces market prices (see Figure A1 in the *Appendix*). Extra-profits turn even negative in the medium to long run. In addition, the *general rate of profit declines for every level of the exploitation rate*. Employed workers may well enjoy higher real wages due to the higher labour productivity, hence the lower market prices (Figures A2, A3 and A4). However, there is no improvement for the working class considered as a whole. Income losses of the unemployed exactly match income gains of the employed.[6]

B) *Individual-based definition of subsistence (or sticky real wage rate).* It is now assumed that I-sector capitalists can capture a large share (50 percent on average) of productivity earnings. This seems a more realistic assumption after all. Under an unregulated free-market capitalist economy, workers can hardly capture the productivity gains, because of the pressure made by the industrial reserve of army and the lack of institutional protections for both the employed and the unemployed. Under a more regulated system (e.g. a social democracy), the share of productivity gains that eventually goes into the coffers of the wage-earners may well be much higher. However, it is still unlikely to be unity.

The key findings are listed below:

- If the real wage rate is defined in terms of individual subsistence, the class exploitation rate is expected to increase when labour-saving innovations are introduced, thereby supporting (rather than depressing) the general rate of profit realised by the capitalist class.

- More precisely, if the capitalists get a large share of the productivity gains, there is a tendency for the profit rate to increase, rather than to decrease, following innovations. This is shown by the blue line in Figure 8, which marks the average profit rate when the capitalists enjoy a positive share of productivity gains. The two dotted lines mark the upper and the lower limits, respectively.[7] The green area highlights the scenarios (Monte Carlo simulations) characterised by a rise in the profit rate. By contrast, the red area highlights the scenarios in which the profit rate falls as the innovation spreads.

- The reason why the profit rate grows is that the percentage increase in the *ex post* class exploitation rate (Figure 9) outstrips the increase in the organic composition of capital (Figure 10). In the proposed numerical simulations, the average growth rate of the former is 10 percent, while the growth rate of the latter is approximagely 9 percent.

- If the capitalists only get a small share of the productivity gains, the profit rate increases for the innovators but not for the routine capitalists and, on average, the capitalist class as a whole.

- Besides, routine capitalists' and late-innovators' profits are expected to be affected anyway. The idiosyncratic trends for the profit rates realised by different groups of capitalists are displayed by Figure 11. While labour-saving innovations are expected to be convenient for the innovators (if the share of productivity earnings taken by the workers is low enough), they are usually detrimental for the system-wide profit rate.

- An implicit corollary of the proposed simulations is that the general profit rate is influenced (also) by the degree of *contestability* of the markets. The higher this degree, the more rapidly the effect of each innovation spreads throughout the system. Therefore, market non-contestability can act as a stabilising factor (acting like a ship compartment) for capitalists' profits. In fact, it can be regarded as an *additional counteracting force* keeping the general profit rate from falling.

[6] Real supplies are unchanged instead (apart from stochastic variations). This should not come as a surprise, for it has been assumed that the rise in labour productivity, enabled by the new technique of production, exactly offsets the cut of labour inputs (see Figures A5 and A6).

[7] That is, the average profit rate $\pm$ 2 standard deviations.

Fig. 8 - General profit rate: individual subsistence following innovation in I-sector

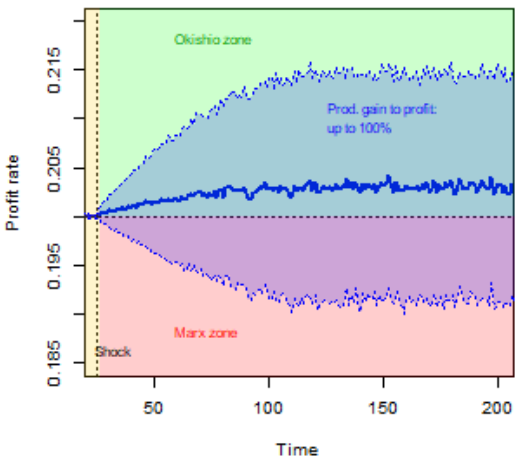


Fig. 9 - Actual exploitation rate: individual subsistence following innovation in I-sector

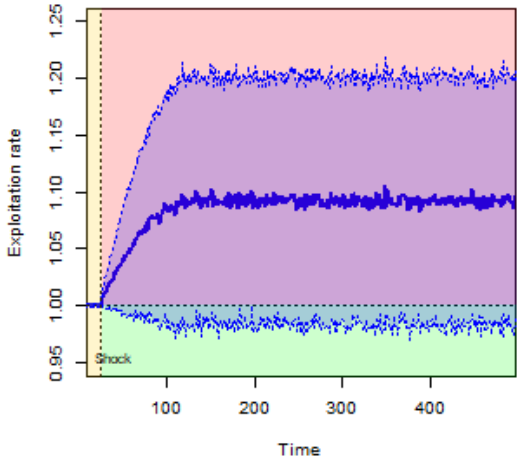


Fig. 10 - Actual OCC: individual subsistence following innovation in I-sector

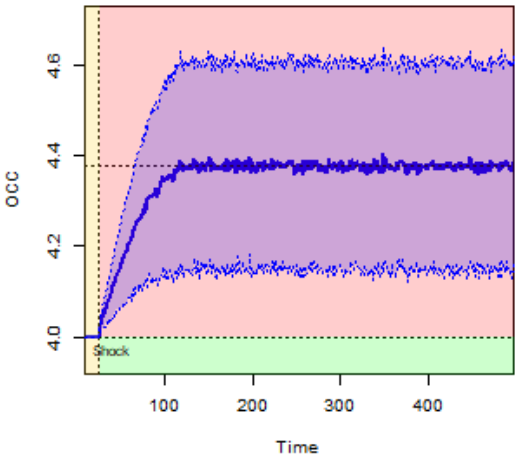
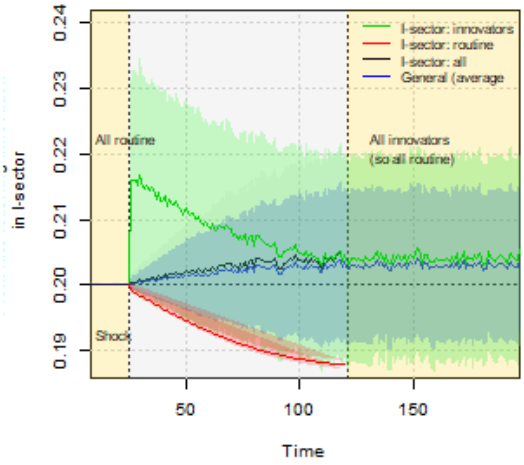


Fig. 11 - Profit rates: innovation in I-sector (individual subsistence)



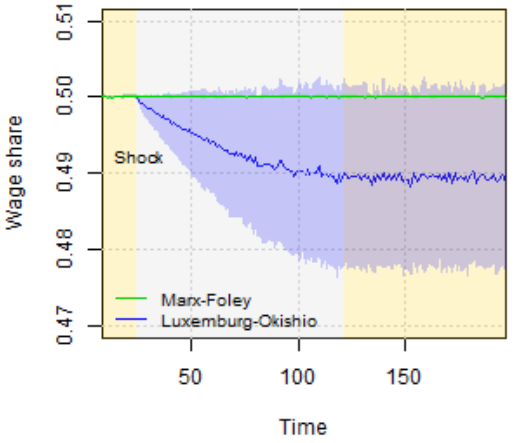
- Another corollary is that labour-saving innovations can endanger working-class reproduction conditions if individual subsistence is defined in terms of *primary needs*. Now the fall in the real wage bill (hence in the wage share) is likely to trigger a response from the workers. This point, which should be analysed thoroughly in future works, can be regarded as a class-struggle foundation of the assumptions made in sub-scenario A.

Notice that the proposed numerical exercises allow shedding light on the relevance of the Okishio's Theorem (OT). Okishio (1961)[9] holds that, if the individual real wage is constant, the general profit rate must rise following the introduction of a *viable* technique of production, that is, a new technique that cuts production costs. The OT has given raise to a long-last controversy since its formulation. While many criticisms and defenses have been made, the findings discussed above are coherent with Foley's critical rendition of the OT. If the *value of labour-power* (not the individual real wage) remains constant, the general profit rate falls with the new technique (Foley 1986, p. 151).[4] This is tantamount to assuming that the capitalists do not benefit (much) from productivity gains, so that the wage share remains (approximately) constant.[8] Since real wage rates have been growing (on average),

[8] The value of labour power is the social labor time that the working class receives in exchange for a unit of labor-power. In formal terms, it can be defined as: $v_{ij} = V_{ij}/L_{ij}$, where V_{ij} is the variable capital

while the value of labour power has been falling in the last decades, it is impossible to assert *a priori* whether the general profit rate should be increasing or falling in the real world. However, the model proposed shows that it can be convenient for some capitalists (the innovators) and necessary for other (the late-comers) to keep introducing labour-saving technologies and methods, even though this can be associated with a fall in the general profit rate. Consequently, while Okishio-like results can be observed both in the real world (over certain periods) and in our experiments (for some specifications of model's coefficients) they are not generalisable.

Fig. 12 - Wages to net income ratio following innovation in I-sector



5 Final remarks

An agent-based RS model has been developed to revisit Marx's theory of profit. The new model helps address three interconnected questions: does the original *tendency for the profit rate to fall* story hold? What is the impact of labour-saving innovations? What is the significance of the Okishio's theorem? The key finding is that the Okishio' theorem is no longer generalisable once the assumption of a *representative agent* is replaced with an *heterogenous agent*-based model. For the introduction of labour-saving innovations, driven by the search for higher individual profit rates, is consistent with both a rise and a fall in the general rate of profit. In fact, Marx's and Okishio's findings can be re-thought as the two poles of a spectrum of tendencies for the general profit rate, mainly driven by the specific dynamics of real wages.

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[1] Bronfenbrenner, M. (1973) "The Marxian MacroEconomic Model: Extension from Two Departments", *Kyklos*, 26(4), pp. 201-18.

[2] Cockshott, W. P. (2016) "Marxian Reproduction Prices Versus Prices of Production: Probability and Convergence", University of Glasgow, unpublished working paper.

[3] Duménil, G. and Foley, D. (2008) "The Marxian Transformation Problem', in S. N. Durlauf and L. E. Blume (eds.), *The New Palgrave Dictionary of Economics*, 2nd edition (Basingstoke: Palgrave Macmillan).

[4] Foley, D. (1986) *Understanding Capital: Marx's Economic Theory* (Cambridge, Massachusetts: Harvard University Press).

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(which matches the necessary labour time in our model, because the monetary expression of the labour time is unity) and L_{ij} is the amount of social labour time spent in the production process throughout a period of time. It is easy to show that, if an individual-based definition of subsistence is used, the value of labour power falls (that is, class exploitation rises) following a labour-saving innovation. This assumption was made, among others, by Rosa Luxemburg. By contrast, if a class-based definition of subsistence is used, the value of labour power and the exploitation rate remain constant (see Figure 12).

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[7] Marx, K. (1894) *Capital: A Critique of Political Economy*, Volume III (London: Penguin Books, 1981).

[8] Morishima, M. (1973) *Marx's Economics: A Dual Theory of Value and Growth* (Cambridge: Cambridge University Press).

[9] Okishio, N. (1961) "Technical change and the rate of profit", *Kobe University Economic Review*, 7(1), pp. 85-99.

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A Appendix. Prices and output

Fig. A1 - Market price of I-goods following innovation in I-sector

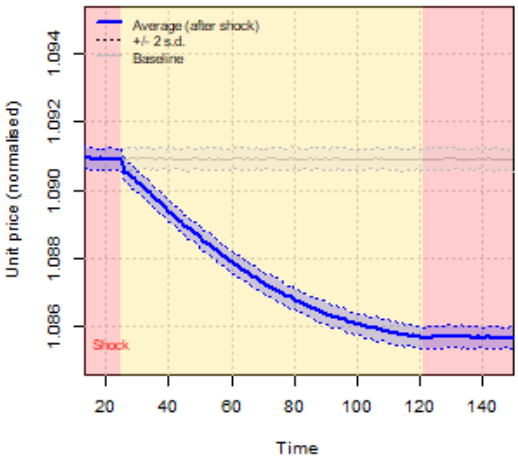

Fig. A2 - Market price of C-goods following innovation in I-sector

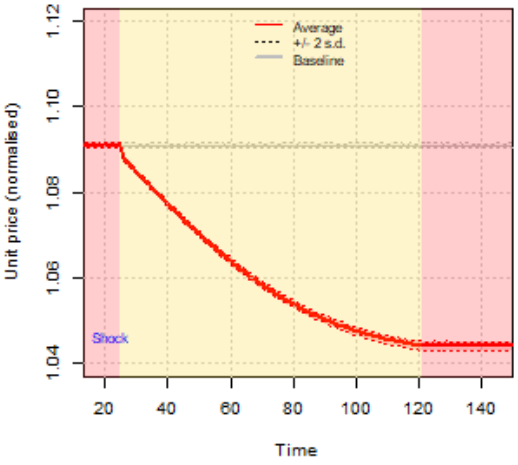

Fig. A3 - Labour productivity in I-sector following innovation in I-sector

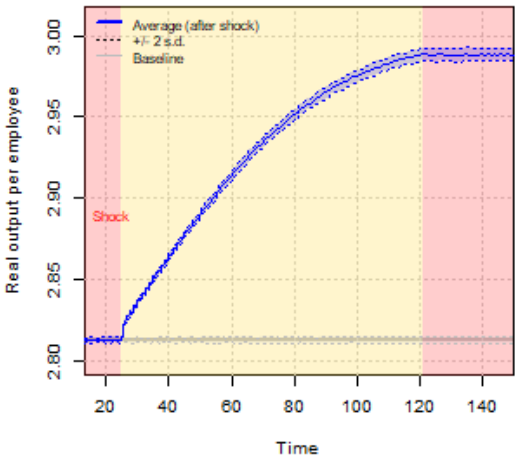

Fig. A4 - Real wages received by employees following innovation in I-sector

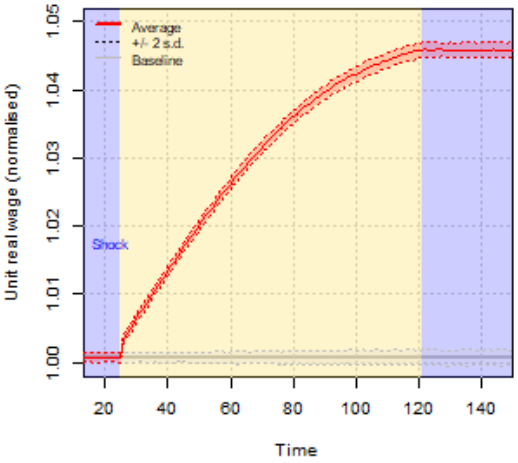

Fig. A5 - Real supply of I-goods following innovation in I-sector

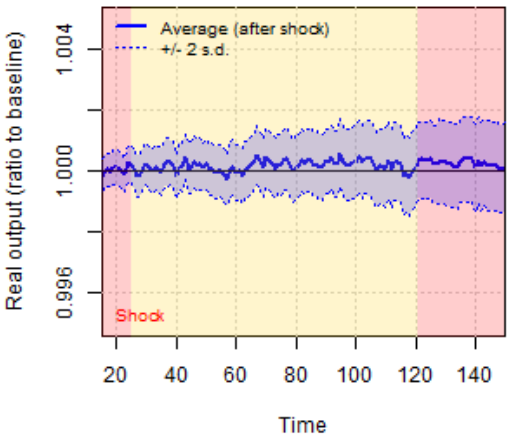

Fig. A6 - Real supply of C-goods following innovation in I-sector

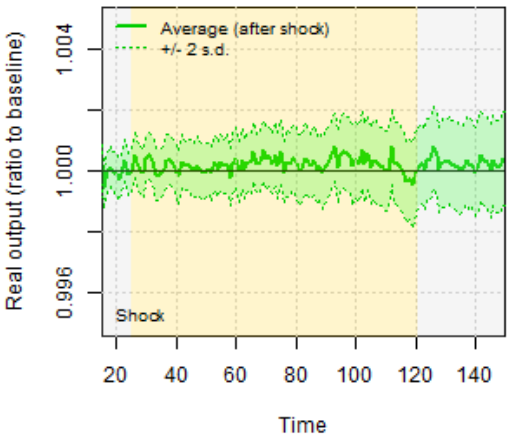

Fig. A7 - Percentage of innovators