Stock-Flow Consistent Dynamic Models: Features, Limitations and Developments

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Abstract: The stock-flow consistent (SFC) approach to macroeconomic dynamic modelling was developed in the 2000s by Godley and Lavoie, (2007a, 2007b), who paved the way for the flourishing of SFC models. These models are based on four accounting principles (flow consistency, stock consistency, stock-flow consistency, and quadruple book-keeping), which allow inferring a set of accounting identities. The latter are then coupled with a set of equations defining the equilibrium conditions. Finally, difference (or differential) stochastic equations are added to define the behaviour of each macro-sector (or agent) of the economy. SFC models' coefficients can be calibrated to obtain a theoretical baseline scenario and/or estimated through standard econometric techniques. Baseline results are then compared with a variety of 'possible worlds' or shocks. This theoretical and analytical flexibility is the reason SFC models are used by economists with different theoretical backgrounds. While SFC models are affected by some limitations, we believe that advantages outdo weaknesses.

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1. Introduction

The stock-flow consistent (SFC) approach to macroeconomic dynamic modelling was developed in the 2000s by Godley and Lavoie (2007a, 2007b), who paved the way for the flourishing of SFC models. These models are based on four accounting principles (flow consistency, stock consistency, stock-flow consistency, and quadruple book-keeping), which allow inferring a set of accounting identities. The latter are then coupled with a set of equations defining the equilibrium conditions. Finally, difference (or differential) stochastic equations are added to define the behaviour of each macro-sector (or agent) of the economy. SFC models' coefficients can be calibrated to obtain a theoretical baseline scenario and/or estimated through standard econometric techniques. Baseline results are then compared with a variety of 'possible worlds' or shocks. This theoretical and analytical flexibility is the reason why SFC models are used by economists with different theoretical backgrounds. While SFC models are affected by some limitations, due to both their 'heavy' structure and the ex-post book-keeping they are based on, we believe that advantages outdo weaknesses. In this chapter, we provide a survey of SFC models' literature and we discuss a possible new research strand, presenting an experimental two-country SFC ecological model.

The rest of the chapter is organised as follows. Section two outlines the origin of the SFC approach. Section three presents and defines the main features of modern SFC models, which are compared to standard DSGE models in section four. Specifically, we focus on model linearity issues, parameter estimation methods, types of micro-foundations, and the intelligibleness of model outcomes, respectively. Section five deals with recent developments in SFC modelling. While many successful attempts at cross-breeding the basic model with other approaches (e.g. agent-based and input-output models) have been made in the last decade, we focus on two promising 'internal developments': multi-country SFC models and ecological SFC models. In section six, we present an ecological two-country model prototype. We show that, if household consumption plans are sensitive to climate change (and/or other natural phenomena), balances of payments and government budgets of less ecologically-efficient countries or areas are affected. This, in turn, can trigger a reaction (either in form of austerity measures or protectionist policies) that ends up reducing the international volume of trade and world output. While green innovation slows down, the fall in world output can bring about beneficial effects for the environment. However, in principle, a 'high road' to ecological sustainability would be also possible if a coordinated macroeconomic plan, aiming at fostering green innovation, was negotiated by the two parties. Concluding remarks are provided in section seven.

2. Origins of the SFC approach

Arguably, the early theoretical roots of the SFC approach trace back to the works of Keynes (1936) and Kalecki (1971). However, it is the pioneering work of Copeland (1949) that provided the methodological cornerstone SFC models are built upon (e.g. Caverzasi and Godin 2015, Nikiforos and Zezza 2017). Copeland (1949) integrated the standard national income identity with the flow of funds through the quadruple accounting principle, thus establishing a simple and sound method to link economic and financial flows with stocks. The bridge between the Keynesian macroeconomic theory and Copeland's (op. cit.) methodology was later provided by Tobin (1981) and his research group, based at the Yale University in the New Haven (hence the name 'New Haven School' attributed to it). In his Nobel Memorial Lecture, Tobin (1982) stressed that the task of the economists is "to bring the columns [of the flow of funds account] to life by functions relating sectorial portfolio and saving decisions to relevant variables, and to bring the rows to life as a set of simultaneous market-clearing equations" (p. 16). To bring the columns to life means, for instance, to link households' wealth-allocation decisions across different financial assets to their relative return rates, market price expectations and agents' liquidity preference. The use of Tobinesque principles to define portfolio equations is one of the key features of SFC models. Tobin's adding-up (or vertical) constraints for portfolio coefficients (Tobin 1969) have been integrated with additional horizontal constraints (Godley 1996) to make financial assets' demand functions fully consistent. Similarly, to bring flow of funds matrix's rows to life means specifying the mechanism that allows matching the demand for financial assets with their supply. This also enables assuring the system-wide equality between financial assets and financial liabilities. While market-clearing prices are usually used for the equity and shares market and other securities, SFC modellers are reluctant to extend price adjustment mechanisms to other markets.

Tobin's (1982) influence on the SFC community is not confined to the use of portfolio equations. In fact, his Memorial Lecture has become a sort of 'manifesto' for SFC modellers. The main points can be summarised as follows:

a) *Precision regarding time*. Models must be dynamic, meaning they must evolve through non-ergodic (discrete) time, in which the economy's state at a certain time depends on the previous periods' states.
b) *Tracking of stocks*. Stocks change over time by virtue of flows, and the whole system is affected by their feedback on transactions-flows (e.g. consumption, investment and production decisions).
c) *Plurality of assets (liabilities) and return rates*. There are many different financial assets (liabilities) in the system, each of which is possibly characterised by a different interest rate.
d) *Integration of real and financial sides*. Not only is the financial sector made up of a plurality of financial assets in addition to cash money, but there is no long-run neutrality of money.
e) *Adding-up constraints and Walras' law*. As mentioned, portfolio equations must be subject to algebraic constraints to meet the stock-flow consistency criteria. In addition, for the Walrasian principle one equation of the model is logically implied by all the others and must be dropped to avoid over-determination. In fact, it can be used as a test to check the consistency of the model.

While Tobin's speech did not have a major impact on the Neo-Keynesian School (which was struggling with the 'rational expectations' revolution), his theoretical baton was taken over by the Cambridge Economic Policy Group (henceforth, CEPG), led by Wynne Godley. In his *Background memories*, Godley explains this point as follows:

> I remember the damascene moment when, in early 1974 (after playing round with concepts devised in conversation with Nicky Kaldor and Robert Neild), I first apprehended the strategic importance of the accounting identity which says that, measured at current prices, the government's budget deficit less the current account deficit is equal, by definition, to private saving net of investment. Having always thought of the balance of trade as something which could only be analysed in terms of income and price elasticities together with real output movements at home and abroad, it came as a shock to discover that if only one knows what the budget deficit and private net saving are, it follows from that information alone, without any qualification whatever, exactly what the balance of payment must be. (Godley and Lavoie 2007a, pp. xxxvi-xxxvii)

The line of research pursued by the CEPG (e.g. Godley and Cripps 1983; Coutts et al., 1985; Godley and Zezza, 1989) shows clear resemblance to the one pursued by Tobin and his group (e.g. Backus et al., 1980). This methodological resemblance took the form of a direct collaboration in 1984, when the CEPG invited Tobin to Cambridge to present his portfolio asset allocation approach. The latter has been incorporated in SFC models ever since. The major difference between the New Haven school and the SFC one is theoretical, as SFC theorists have incorporated Tobin's portfolio approach "into a monetary production economy where the supply of money is endogenous and where behavioural equations respond to Kaleckian or Keynesian precepts rather than neoclassical ones" (Lavoie 2014, p. 264). More precisely, SFC models recognise both the long-run impact of changes in aggregate demand on real variables and the independence of investment plans from saving decisions.

In the 1990s Godley joined the *Levy Economics Institute of Bard College*, where he kept refining his approach. Several empirical models for short- and medium-run forecasting were developed, based on SFC principles (e.g. Godley and Zezza, 1992; Godley, 1999; Godley, 1999). These efforts culminated in the development of an empirical model for the US economy, named the 'Levy model' (see Caverzasi and Godin, 2015; and Nikiforos and Zezza, 2017, for a thorough description of it). The Levy model allowed Godley and his group to forewarn about the Dot-Com Crisis of 2000-2002 and the Global Financial Crisis of 2007-2009. The predictive power of the model was recognised by the press and earned Godley's approach an increasing popularity among the practitioners. Arguably, the impact on the academia, particularly on the current mainstream in macroeconomics, was less dramatic. However, the SFC approach has been increasingly considered a "way of unifying all heterodox macroeconomists" (Lavoie, 2014, p. 264). In a sense, an 'alternative consensus' in non-neoclassical macroeconomics has been established, based on the SFC approach, as opposed to the so-called 'new consensus' in mainstream macroeconomics, based on its rendition of the 'dynamic stochastic general equilibrium' (DSGE) class of models. In fact, the stock-flow consistency of models is now regarded as a *conditio sine qua non* for publication by many heterodox economics journals.

3. SFC models: definition and main features

The use of the label 'stock-flow consistent' is quite controversial. It was popularised and became a sort of 'brand' after the publication of Dos Santos (2003)'s definitory work (e.g. Nikiforos and Zezza**,** 2017). Nonetheless, it is still regarded as misleading by some SFC theorists. Significantly enough, it is never mentioned in the 'Bible' of SFC modellers, namely, by Godley and Lavoie (2007a). The fact is that, while some standard macroeconomic models are not stock-flow consistent (think of the traditional IS-LM-AS model), others are (e.g. most DSGE models). As a result, the label 'stock-flow consistent' does not allow to separate models based on the 'neoclassical dichotomy' from models based on a thorough integration of real and monetary aspects. Since the SFC community explicitly rejects the neoclassical dichotomy, a different label is sometimes invoked to avoid confusion. However, a new label could possibly foster controversies and misunderstandings, rather than smoothing them out. Consequently, we stick to the standard definition hereafter.

Narrowly-defined SFC models are based on four accounting principles (e.g. Nikiforos and Zezza, 2017; Deleidi et al.**,** 2018):

a) *flow consistency*, meaning that every transaction-flow must come from somewhere and go somewhere;
b) stock consistency, meaning that the financial liability issued by an economic unit (be it a firm, a household, a bank, a financial intermediary or the State) must be held as a financial asset by another economic unit;

c) *stock-flow consistency*, meaning that flows affect stocks and this impact must be accurately registered (including capital gains and losses);

d) *quadruple book-keeping*, meaning that every transaction requires filling in four different entries.[1]

These four principles are incorporated in the Balance-Sheet (BS) and the Transactions-Flow Matrix (TFM) of the economy, providing the accounting framework SFC models' identities are derived from. The BS encompasses sectoral assets and liabilities. Assets are usually recorded using a positive sign, while liabilities (and net worth) have negative sign. The TFM is obtained by combining the national income equation with the sectoral flow of funds accounting. Receipts or sources of funds are usually recorded using a positive sign, whereas payments or uses of funds are given a negative sign. The latter displays the changes in the stocks at time t due to purchases of new assets (issues of new liabilities). Notice that changes in stocks' values due to changes in assets' prices are not included here. They are sometimes recorded as revaluation effects in a third matrix, named the Full-Integration Matrix (FIM), where each sector's net wealth at time *t* is calculated by adding capital gains (subtracting capital losses) to net wealth at time *t* – 1.

SFC matrices, particularly the TFM, allow inferring the first set of model equations in form of accounting identities (e.g. national income, net wealth, etc.). The latter are then coupled with a second set of equations defining the equilibrium conditions (e.g. labour supply equals labour demand). Finally, difference (or differential) stochastic equations are added, to define the behaviour of the macro-sectors of the economy under observation (e.g. consumption, investment and import functions). The inclusion of behavioural equations, which are usually borrowed from the post-Keynesian tradition in economics, differentiates SFC models from purely 'hydraulic' models, which just rely on accounting principles. Identities only provide a general set of constraints, which enable reducing the 'degrees of freedom' of models. In fact, early SFC modellers thought that building models upon a sound accounting structure would have reduced significantly the range of possible long-run findings (e.g. Godley and Cripps, 1983). Unlike Solow-type models, SFC models do not have their medium-run dynamics constrained by any supply-side exogenous attractor (e.g. the 'natural output' and/or the 'natural unemployment level'). Production and employment are always demand-led.[2] Full employment is not guaranteed by price flexibility. The economy's medium-run dynamics is constrained by the accounting structure of the model. If a sector or a national economy is running a surplus, there must be another sector or national economy that is facing a deficit, after all. A policy corollary follows that fiscal policies and/or other types of intervention of the government sector are necessary to achieve and maintain full employment and financial stability, while traditional monetary policies are usually less effective. This is not to say that stochastic equations are uninfluential in the modelling. On the contrary, behavioural hypotheses are still crucial, "as has been confirmed when new SFC models, with assumptions

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[1] More specifically, there must be always an inflow in favour of a unit, call it A, that matches the outflow faced by another unit, call it B, along with a reduction in assets held by (or an increase in liabilities of) unit A that matches the increase in assets held by (or the reduction in liabilities of) unit B.

[2] However, the accumulation of (unsold) inventories is possible when actual demand falls short of expected demand and hence firms' production plans turn up to be too optimistic. In addition, credit rationing is considered, and supply-side constraints may well arise from the ecosystem (e.g. climate change and the depletion of natural reserves of matter and energy). The central role played by aggregate demand is the reason some authors refer to these models as 'post-Keynesian stock-flow consistent' models (e.g. Caverzasi and Godin, 2015).

slightly different from those of the earlier ones, produced different trajectories" (Lavoie, 2014, pp. 273-274).

Focusing on one-country SFC models, four sectors are usually considered, notably, households, firms, banks, and the government. However, SFC models can be further extended to include additional sub-sectors (e.g. wage-earners as opposed to rentiers and capitalists, non-bank financial institutions as opposed to commercial banks, etc.), national economies, and other social and ecological variables. Once their theoretical structure is set up, SFC models are usually solved through computer numerical simulations.[3] For this purpose, models' coefficients can be:

a) calibrated, based on stylised facts or rules of thumb;
b) estimated through standard econometric techniques;
c) fine-tuned in such a way to obtain a specific baseline scenario.

Arguably, option (a) is still the most popular. In this case, the robustness of the results is usually checked through sensitivity tests. However, option (b) has gained momentum in the last decade, as empirically estimated models are more suited to policy purposes. Method (c) is employed to set coefficient values that cannot be estimated or to obtain a specific baseline. These values are often calculated using model equations along with observed initial values for stocks and lagged endogenous variables. Finally, auto- and cross-correlation structures of simulated data (or out-of-sample predictions) are sometimes compared with the observed ones. The aim is to verify whether observed and simulated series share the same statistical properties.

Whatever the calibration method chosen, baseline results are usually compared with a variety of scenarios or shocks. SFC models' analytical flexibility is the reason they are used by economists with different theoretical backgrounds. In fact, SFC models have been crossbred with other non-neoclassical approaches, including 'interacting heterogeneous agents'-based models, input-output analyses, supermultiplier mechanisms, ecological flow-fund models, etc. Despite this heterogeneity, narrowly-defined SFC models are all based on a sound macroeconomic accounting, which allows for a complete integration of the real and the financial side of the economy. Money is not a veil laid over real variables. On the contrary, real variables are affected by the way credit money is created, circulated and destroyed in the economy. The money creation process, in turn, is affected by real variables (for instance, an increase in the expected growth rate of output raises the level of investment that can be financed by bank loans). Government money (or high-potential money) is also considered, along with a variety of financial assets and liabilities (including loans, mortgages, deposits, bonds, shares & equities, other securities, and derivatives). For SFC models are not just stock-flow *consistent*, but also stock-flow *relevant*, meaning that are built upon a 'realistic' description of how a financially-sophisticated capitalist economy works.

4. SFC vs. DSGE models: a comparison

Dynamic stochastic general equilibrium (DSGE) models have dominated mainstream macroeconomics in the last three decades. Focusing on the basic (closed-economy) reduced-form model, there are usually three theoretical building blocks, which define an IS-like curve (demand side), a Phillips curve (supply side) and a monetary rule, respectively.[4] Each block

[3] Algebraic or analytical solutions are sometimes provided for the simplest models but cannot be calculated for the most sophisticated models.

[4] Also the simpler, benchmark 3-equation model, also known as the New Consensus Model, is built upon an IS curve, a Phillips curve and some sort of a monetary rule. See for example Lavoie (2015).

is micro-founded, meaning that each equation of the reduced-form macroeconomic model is obtained by solving a problem of intertemporal maximization subject to constraints. More precisely, to obtain the IS-like curve, it is assumed that the representative household maximizes its lifetime utility function (including consumption, real money balances and hours worked, plus the discount factor, labour supply elasticity, and other 'deep' parameters) subject to a budget constraint. To obtain the Phillips curve, it is assumed that the representative firm maximizes its profit subject to the technical constraints (production function) and/or price constraints (e.g. menu costs). To derive the monetary rule, it is usually assumed that the central banker steers the money market interest rate in such a way to minimize its loss function (whose arguments are the deviation of inflation from the target level and the deviation of current output gap from the natural level) subject to the Phillips curve. DSGE models can be regarded as a particular class of Real Business Cycle (RBC) models (e.g. Kydland and Prescott, 1982; Long and Plosser, 1983). Therefore, it is no surprise that early RBC-DSGE models were based on the assumption of perfect markets, entailing price and wage flexibility. Since the mid-1990s, these models have been gradually displaced by a new generation of DSGE models, named New Consensus Macroeconomics (NCM) DSGE models. Like the original RBC-DSGE models, NCM-DSGE models usually rely on rational expectations. However, NCM-DSGE models account explicitly for market imperfections (rigidities, frictions and asymmetries) in the short term, which can temporarily keep current output from adjusting to its natural or long-run equilibrium level. However, price stickiness and other imperfections also allow the central bank to influence the real interest rate (and so current output and inflation) in the short run, while steering the nominal rate. Besides, NCM-DSGE models can include non-microfounded parts (e.g. the backward-looking component of inflation in the so-called NCM Phillips Curve), based on empirical evidence, to improve their fit of observed time series.[5] The models are then calibrated or estimated (using Bayesian techniques) in such a way to trace the behaviour of macroeconomic variables back to 'deep structural parameters' defining agents' preferences and technical and institutional constraints.

Despite the relatively higher degree of 'realism' (compared to the early DSGE models), NCM-DSGE models have been harshly criticised in the aftermath of the Global Financial Crisis. In fact, their use is still highly contentious (e.g. Mankiw, 2006; Roemer, 2016; Stiglitz, 2018; Krugman, 2018). Unsurprisingly, SFC models are frequently mentioned as possible alternatives (e.g. Burgess et al., 2016; Caiani et al., 2016). However, are SFC models exempt from the flaws attributed to NCM-DSGE models? To address this question, we focus on four standard criticisms directed at mainstream models, which concern model linearity, parameter estimation methods, types of micro-foundations, and the intelligibleness of model outcomes, respectively.

As mentioned, DSGE models (be they RBC or NCM) usually assume rational expectations, that is, economic agents use all available information and know the model underlying the economy. As a result, agents never make systematic errors. Predictions are correct on average, meaning that they do not differ predictably from equilibrium results. The use of rational expectations allows *inter alia* justifying the stability of models, thereby making the linearization of nonlinear economic systems possible. Typically, DSGE models' solutions are all unstable except one, that is, the 'saddle path solution' (e.g. Rankin, 2011). The existence of a unique and stable equilibrium is guaranteed by the so-called 'transversality condition',

[5] Allegedly, the most famous DSGE model is the so-called Smets-Wouters model, developed by the European central bank (Smets and Wouters, 2003; see also Lindé et al., 2016).

which, in turn, is based on the rational behaviour hypothesis.[6] If the latter were abandoned, the former would be hardly met. Notice that linearity, as an assumption, is what allows the models to extrapolate existing trends from time series to predict future values. While this may sound controversial, SFC models are usually based on linear equations too. However, they can be amended to incorporate non-linearities. Besides, while DSGE models are specifically used to extrapolate existing trends into the future, SFC models are generally used to ask whether existing trends can be sustained (Keen, 2016).

A second alleged flaw of DSGE models concerns the estimation of model coefficients. Both calibration and Bayesian estimation techniques have been criticised. The problem with the former is that 'the choice to rely on a 'standard set of parameters' is simply a way of shifting blame for the choice of parameters to previous researchers' (Blanchard, 2018, p. 45). As for the latter, 'the justification for the tight priors is weak at best, and what is estimated reflects more the prior of the researcher than the likelihood function' (*Ibidem*). Once again, SFC models are possibly affected by the same problem. An accurate estimation of parameter values can improve models' fit. However, it is hard to understand whether the model is actually capturing any 'causal trend' or it is just reproducing the noise in the data instead. To reduce the risk of pure data-fitting exercises, SFC modellers usually opt for equation-by-equation estimation techniques (sometimes based on cointegration). In addition, the inclusion of financial assets (liabilities) stocks and flows, along with the accurate modelling of different sectors (including the financial sector), make SFC models more realistic and, arguably, less likely to blunder. It is sometimes counter-argued that DSGE models have incorporated some financial variables, and even some agent heterogeneity, since the mid-2000s. However, DSGE models simply treat these features as additional sources of frictions, which can only slow down the process of convergence to a predetermined, natural, equilibrium. Unlike IS-LM-AS models, DSGE models are usually stock-flow consistent, as money is defined residually. However, they are not stock-flow relevant, as financial stocks and flows are like sand in the machine, rather than a crucial gear of it.

A third issue with DSGE models concerns the type of microfoundations they are based on: a representative agent maximising her utility or profit function subject to constraints.[7] This is said to address the Lucas critique about the sensitivity of macroeconometric models' parameters to changes in the policy stance. For calibrating or estimating deep structural parameters (preferences, technical constraints, etc.) would allow anchoring the model to invariant magnitudes. However, three major counter-arguments can be raised here. First, the empirical relevance of the Lucas critique has been questioned (e.g. Smith, 2009). Second, deep structural parameters are not as invariant as they are assumed to be (Altissimo et al., 2002). Third, linking the dynamics of a macroeconomic system to the behaviour of a representative agent is a smart *escamotage* to address the well-known flaws of early Walrasian general equilibrium models (concerning the uniqueness and stability of the equilibrium). However, capitalist economies are complex systems. The latter show emergent properties that their individual parts do not possess, and which result from the interaction of

[6] The transversality condition rules out explosive paths or bubbles, when the current value of a certain variable, say the inflation rate, depends on its expected future value. It holds that the increase in expected inflation is not 'too fast'. As a result, the path of inflation is convergent.

[7] As mentioned, some agent heterogeneity has been allowed for in the last decade. For instance, Ricardian households (who can borrow and lend to smooth their consumption over time) are now sometimes coupled with non-Ricardian households (who cannot rely on the credit market). However, this is just a different type of friction, which in no way affects the qualitative behaviour of the model in the long run.

the individual parts. This means that the behaviour of aggregate variables must be studied either from a macroeconomic perspective (i.e. macrofoundations) or from a bottom-up generative approach (i.e. interacting agents-based microfoundations). SFC models are suitable for both options, whereas DSGE are stuck in a theoretical limbo.

The main advantage of DSGE models, compared to SFC models, seem to be the simplicity of the narrative brought about by the former. Technically speaking, DSGE models are highly sophisticated models. Their developers are required to master both dynamic optimisation techniques and Bayesian statistics. Nonetheless, once a model is set up, it produces '(at least when linearized) a VAR representation of the endogenous variables that should, in theory, be straightforward to take to the data' (Burgess et al., 2016, p. 3). In addition, reduced-form three-equation DSGE models are very useful to tell the students, the practitioners, and the policy makers, a simple story about the way our economies work. The implied causality is simple and intuitive: the central bank steers the nominal interest rate, thus affecting the real cost of money (because prices are sticky in the short run) and hence current output; the latter, in turn, determines the inflation rate, in the long run, which is also influenced by price expectations. This is the reason why central banks must be credible (mainly inflation-targeting) institutions, independent from the control of governments and parliaments. By contrast, the interpretation of SFC models' outcomes is not always straightforward. On the one hand, the transactions-flow matrix includes a variety of flow variables, whose interactions (and the interaction with stock variables) determine model dynamics. The latter is not trivial, as there is no long-run attractor that predetermines it. As a result, multiple equilibria are possible. On the other hand, the System of National Account (SNA) is far more detailed than any 'tractable' SFC model could possibly be. For instance, the ONS Blue Book contains around 6,500 time series referred to transactions and other flow variables, while an SFC model usually includes less than 200 series. Unfortunately, there is still no standard method to match the usual SFC matrices with the information provided by the SNA, even though a few attempts have been made in recent times (e.g. Burgess et al., 2016; Veronese Passarella, 2019).

5. Recent developments in SFC modelling

The SFC approach has been developed in a period marked by two major economic and financial crises of advanced countries. SFC models are naturally fit for the analysis of the interaction between the real and financial sector. So, it is no surprise that the so-called 'financialisation' process has been one of the most popular topics in the SFC literature since its inception (e.g. Skott and Ryoo, 2008; Lavoie, 2008; van Treeck, 2009; Hein and van Treeck, 2010; Michell and Toporowski, 2012; Morris and Juniper, 2012; Veronese Passarella, 2012; Reyes and Mazier, 2014; Botta et al. 2015; Sawyer and Veronese Passarella, 2017). The increasing importance of financial motives, tools and agents is not the only subject covered by SFC theorists. Income distribution, credit rationing, growth determinants, economic policies and ecological issues have been covered as well (for a detailed rendition, see Caverzasi and Godin, 2015, and Nikiforos and Zezza, 2017). In addition, there have been at least two types of 'external' development, or cross-fertilisations, and three types of 'internal' development of SFC models.

The former aim at crossbreeding the original model with other analytical tools and/or modelling techniques. Successful cross-fertilisations include agent-based SFC models (AB-SFC) and input-output SFC models (IO-SFC), providing the basic model with micro- and meso-foundations, respectively. AB-SFC models are usually employed to detect the emergent properties of the economic system resulting from the interaction of a variety of heterogeneous agents, in which the use of a generative or bottom-up approach is required. These models are particularly fruitful in the study of financial diseases, such as bankruptcy chains, financial contagion phenomena, etc. (e.g. Caiani et al., 2016). Besides, they have been employed to detect the effects of distributive inequality and credit constraints on consumption, investment and output (e.g. Cardaci and Saraceno, 2016; Botta et al., 2018). IO-AB, in contrast, allow analysing the process of dynamic structural change that characterises capitalist economies, which a standard aggregate SFC model would not be able to capture (e.g. Berg et al., 2015).

The three main types of internal developments are: empirical SFC models (E-SFC), open-economy or multi-country SFC models (MC-SFC), and ecological SFC models (Eco-SFC).

5.1 E-SFC models

While the majority of SFC works are still purely-numerical-simulation models, there is an increasing interest for empirical applications of the SFC approach. We can name E-SFC models those in which (most) unknown parameters are estimated from available data through econometric procedures. The most popular estimation methods are equation-by-equation ordinary-least squares (OLS) and vector-error correction models (VECM). Unlike DSGE modellers, SFC modellers rarely opt for system estimation techniques instead. Although the latter may allow for an excellent fit of past data, equation-by-equation methods enable attributing clear economic meaning to behavioural equations' coefficients, thereby helping the modeller to extrapolate and detect the economy's laws of motion. Initial values of stocks and lagged endogenous variables are also set in line with available data.

Focusing on the main characteristics, we can distinguish early E-SFC models, including the Levy model, from those developed in the last decade. Early models are usually developed starting from available data, rather than an already-made theoretical model. In addition, the information they rely on (e.g. data, code, program files) is usually not freely accessible. An example of early E-SFC model is the one used by Godley and Zezza (1992), who applied it to the Danish economy. The Levy model underpins the analyses released by the Levy Institute for the US economy (e.g. Godley 1999; Godley and Zezza 2006; Godley et al. 2007; Godley et al. 2008; Papadimitriou 2009; Papadimitriou et al. 2011; Papadimitriou et al. 2014), and the Greek economy (e.g. Papadimitriou et al. 2013a; Papadimitriou et al. 2013b; Papadimitriou et al. 2015). Most recently, SFC modellers have stressed the need for making all the information available to the public, thus assuring the reproducibility of results. There are also some methodological differences between the Levy approach and other E-SFC models. For instance, in the so-called 'Limerick model' for the Irish economy, most coefficients are not estimated as fixed parameters, but calibrated to fit available time series (e.g. Kinsella and Aliti 2012, 2013; Godin et al. 2012; see also Caverzasi and Godin 2015 on this point). The rationale is to use the model to ask 'as if' questions about alternative policies implemented in the past, rather than to forecast future trends. An analogue approach was used by Miess and Schmelzer (2016a, 2016b) for the Austrian economy. E-SFC models have been also applied to developing countries, e.g. Colombia (see Escobar-Espinoza 2016).

The increasing popularity of the SFC approach has led the Bank of England to develop an E-SFC prototype aimed at providing scenario analyses for the British economy (see Burgess et al. 2016). On the same line, a simple empirical model using Eurostat data for Italy has been also proposed (see Veronese Passarella 2019). In a sense, E-SFC models can be considered a middle-ground between, on the one hand, numerical SFC and other theoretical models and, on the other hand, vector auto-regression (VAR) models.

5.2 MC-SFC models

The analysis of the open economy has been one of most interesting applications of the SFC approach since its launch. The main reference for the 'first generation' of MC-SFC models is chapter 12 of Godley and Lavoie (2007a), where a complete two-country model is developed. It must be noticed that SFC methodology

> differs from the usual textbook approach, according to which models of individual closed economies are eventually 'opened', but which give no consideration to what other countries must be held to be doing and how a full set of interactions between all countries might be characterized. […] We shall discuss open economy macro-economics using models of an economic system which, taken as a whole, is closed, with all flows and all stocks fully accounted for wherever they arise (p. 171).

The model can be run under four different monetary regimes, notably, a flexible exchange regime system, a fixed exchange rate system with foreign reserves, a fixed exchange rate system with an endogenous interest rate, and a fixed exchange rate system with endogenous government spending. The model of Chapter 12, particularly its flexible exchange rate version, has represented the main reference for the subsequent generation of MC-SFC models. When a two-country economy is considered, portfolio equations can take into account the expected change in the exchange rate as one of the factors that drive households' demand for financial assets. This point is developed by Lavoie and Daigle (2011), who integrate the SFC approach with some recent contributions from behavioural finance. More precisely, two different attitudes or behavioural profiles of economic agents (or traders) are considered: a) the 'conventionalists' are those whose expectations are anchored to a conventional long-term value of the exchange rate; b) the 'chartists', in contrast, base their expectations on the previous value of the exchange rate. Main findings can be summarised as follows. First, while expectations are shown not to affect (qualitatively) the trajectories of consumption and investment, they do affect both the long-run level of the actual exchange rate and the trade balance. Second, economic and financial instability increases as the share of 'chartists' to total traders increases. Third, the model exhibits weak hysteresis proprieties, meaning that shocks' effects are more persistent when expectations are considered.

As mentioned, Godley and Lavoie's (2007a) 2-country model has paved the way for the subsequent generation of MC-SFC models. The latter have extended the scope of SFC models beyond their original boundaries. For instance, Ioannou (2018) uses a MC-SFC model to study the impact of credit rating agencies' activity on a two-country monetary union. The model shows that sovereign rating boosts the business cycle, thus accentuating recessionary shocks. As one would expect, the weakest country is particularly affected. In addition, credit rating agencies' 'perception of what constitutes a sustainable debt to GDP ratio have self-fulfilling proprieties and may generate additional instability into the system' (Ioannou, 2017, p. 18).

Nonetheless, the analysis of international imbalances is by far the most popular topic covered by MC-SFC models. In the wake of Lequain (2003), Godley and Lavoie (2007b) developed a three-country model where two of the countries share a currency (i.e. the Euro). Although simulations are purely numerical, the model explicitly mimics the relationship between the US economy and the Euro Area, and the one between peripheral (or deficit) and core (or surplus) members of the monetary union. In hindsight, the model provides some accurate (conditional) forecasts about the effects that the Global Financial Crisis has had on the Euro Area. Using a flexible exchange rate between the US dollar and the Euro, Godley and Lavoie (2007b) show that, when a peripheral Euro Area's member-state faces a current account deficit (following a negative shock to the economy, such as an increase in the propensity to import), there is no automatic readjustment mechanism. For the devaluation of the Euro (with respect to the US dollar) benefits the core member-states, while the periphery keeps running a current account deficit along with a government budget deficit. This means that the policy makers have three options to rebalance the current account (and government budget) imbalances within the monetary union: a) to push the deficit country to cut government spending and/or increase taxes (i.e. austerity measures); b) to persuade the surplus country to adopt expansionary fiscal policies; c) to mutualise the government debt of the Euro Area. Compared to options (b) and (c), option (a) is less effective and entails a remarkable loss in terms of production and employment levels, which seems to be coherent with the historical evidence. In principle, wage cuts in peripheral countries (and/or a wage rise in core countries) can also support the rebalancing process. However, we use a fixed price model that rules out this option.

On the same line, Duwicquet et al. (2012) use a MC-SFC model to test the effect of Euro Bonds and other forms of money transfers from surplus to deficit Euro Area's member-states. The aim is to offset the hidden transfer that runs in the opposite direction because of the exchange rates' misalignment within the Euro Area. Their simulations show that these policies are effective in addressing asymmetrical shocks and rebalancing national current account and government balances. Using a four-country model, Mazier and Valdecantos (2015) analyse four different scenarios. These are: a) the *status quo*, marked by a single currency and floating exchange rates of two Euro Area members (or blocks) with the US and the rest of the world; b) a Eurozone with three 'euros'; c) a return to the European Monetary System, where a 'global euro' is used as an international currency and unit of account; d) a Euro Area without (current) surplus countries. The last scenario is shown to be the most stable. In fact, this solution can be 'beneficial for all' (Mazier and Valdecantos 2015, p. 108). Similarly, Mazier and Aliti (2012) develop a three-country model (including the US, the Euro Area and China) that highlights the negative impact on the Euro Area deriving from a reserves' diversification strategy undertaken by the People's Bank of China (in favour of assets denominated in euros). This confirms the early findings by Lavoie and Zhao (2010). The paper shows also that balance of payments' imbalances are linked with the semi-fixed US Dollar-Yuan parity system. A 'real' floating exchange rate between the US Dollar and the Yuan would help reduce the US current account deficit (and hence the Chinese surplus). Although this scenario is regarded as unrealistic, Chinese authorities could achieve similar results by allowing for a gradual appreciation of the exchange rate in real terms.[8] Finally, using a four country-model, Zezza and Valdecantos (2015) compare a 'US dollar-based' or 'post-Bretton Woods' model with an ideal 'Bancor' model, inspired by the well-known Keynes' plan for the international monetary system. They show that the adoption of a common international currency (i.e. the Bancor) and the establishment of a supranational clearing union would reduce cross-country imbalances, thus helping to achieve economic prosperity and financial stability. The rebalancing effect would result from surplus countries being forced to contribute to the adjustment, as they would be accruing negative interests on their reserve holdings. These payments would be then used to promote investment and innovation in deficit countries.

The main features of a simple two-country model (2C-SFC) are summarised by Table 1 and Table 2, defining the BS and the TFM, respectively. All the variables relevant to the first country or area, named Ecoland, have a g superscript (which stands for 'green'), while all the variables relevant to the other country, named Carbonland, are marked by a c superscript.[9] For the sake

[8] The IMF defines China's regime of exchange rate as 'stabilized arrangements' (FMI, 2017).
[9] The meaning of country names is clarified in section 5.

of simplicity, it is assumed that the private residents of each country do not hold foreign assets.[10] The exchange rate, E, is fixed.[11] As a result, any discrepancy between sales and purchases on the exchange market are made good by transactions of the two national central banks. The latter held all their foreign exchange reserves in the form of gold.[12] The shaded column dividing Table 2 shows that all transactions between the two countries require conversion, because each country has a central bank that issues its own currency. This is accounted for by the BS (Table 1), which also shows that each country has its own government. The latter issues domestic-currency denominated bills. The third row of Table 1 show that each central bank owns a stock of gold reserves. These reserves are physical (not financial) assets. Like fixed capital, they have no liability as their counterpart. As a result, the value of the gold reserves appearing in the last column is not zero, although central banks have zero net worth.

It is assumed that the residents of neither country hold the currency of the other. When they are paid for the products they sell abroad, they exchange their foreign-currency-denominated proceeds into their own currency. Similarly, when they purchase imported products, they must first obtain the foreign currency from the central bank. This means that any 'excess of (private sector) payments for imports over receipts from exports must therefore have an identical counterpart in transactions involving the two central banks, using […] sales or purchases of gold bars valued at some fixed rate in terms of its own currency. With a fixed exchange rate and no restrictions on trade, each central bank must be willing to buy or sell gold on any scale at that fixed rate' (Godley and Lavoie 2007a, p. 189).[13] Finally, an interesting feature of 2C-SFC models' TFMs is that there is no column displaying the balances of payment of the two countries. However, cross-country trade flows can be derived from the two rows describing exports. This is the current account of the balance of payment. Its financial counterpart is given by the changes in (or transfers of) gold reserves, which are a zero-sum game for the economy as a whole.

5.3 Eco-SFC models

An increasing number of either aggregative or agent-based Eco-SFC models have been developed in the last decade, which aim at:

a) detecting sustainable growth conditions and questioning the growth imperative (e.g. Jackson and Victor**,** 2015**;** and Richters and Siemoneit, 2017);
b) studying the energy sector (e.g. Naqvi**,** 2015**;** Berg et al**.,** 2015);
c) investigating the trajectories of key environmental, macroeconomic and financial variables (e.g. Dafermos et al**.,** 2017, 2018);
d) analysing the impact of green fiscal policies and 'green sovereign bonds' (e.g. Monasterolo and Raberto**,** 2018; Bovari et al**.,** 2018);
e) examining the interaction between climate change and financial stability (e.g. Dafermos et al**.,** 2018);

[10] Consequently, one needs not to worry about the exchange rate when summing the elements of each row, except for the stock of gold reserves in the BS, the export (import) entries, and the change in gold reserves in the TFM.
[11] It is here defined as the quantity of Brownland currency in exchange for one unit of Ecoland currency.
[12] These are the key hypotheses underpinning the so-called Model OPEN, i.e. the simplest 2C-SFC model presented in sections 6.6 to 6.9 of Godley and Lavoie (2007a).
[13] This is just a useful modelling simplification. It is well known that gold bars are no longer traded, and reserves of central banks are mainly made up of foreign currencies (US dollars, Euros and other key currencies).

f) or between natural resources' depletion and State-led innovation policies (e.g. Deleidi et al., 2018).

More precisely, Jackson and Victor (2015) raise the question whether growth is necessary for capitalist economies to survive. In other words, they check whether a 'growth imperative' exists, which is determined by the need for the borrowers to pay back the interests due on the stock of outstanding debt. For this purpose, they use a SFC dynamic macro-economic model accounting for the credit creation process led by banks and private equity in a closed economy. They find no evidence of a 'growth imperative'. In addition, they show how an economy can move from a growth to a stationary (or no growing) path. They argue that the countercyclical spending carried out by governments can promote such a transition by smoothing and dampening the oscillations associated with it.

Similarly, Richters and Siemoneit (2017) analyse several SFC post-Keynesian models and question the idea of positive interest rates as the main responsible for the 'growth imperative'. Particularly, a stationary state economy – characterised by zero net saving and investment – is compatible with positive interest rates. The chapter confirms the idea of a debt-based monetary system that does not cause any growth imperative. A stationary state is generated by positive net saving and net investment decisions, which are permanently above zero, and not by a systemic and inevitable necessity.

Naqvic (2015) proposes a multi-sectoral SFC model for a closed economy. Production is demand-led and the economy is made up of several institutional sectors (firms, energy, households, government, and financial institutions), which interplay with the environment. The model is calibrated on the European economy and aims at evaluating the effect of five alternative environmental economic policies (i.e. a de-growth scenario, a capital stock damage function, a carbon tax, a higher share of low-emissions renewable energy, and an investment in technical innovation) on three main challenges (trilemma): (i) boosting output growth; (ii) fostering employment growth with a more equal distribution; or (iii) improving environmental sustainability. The study is motivated by a trilemma that European policy makers are currently facing. Naqvic's (op. cit.) findings show that four out of five policies cannot solve the three challenges simultaneously. Only the investment in innovative technologies can increase output, foster employment (and wage growth), while reduce CO_2 emissions.

Berg et al. (2015) develop a multisectoral ecological SFC model by integrating the flow and stock analysis with the input–output methodology. This allows to model to detect the interaction among three types of flow variables: (i) monetary flows in the financial system; (ii) flows of goods and services produced by the real economy; and (iii) the flow of physical materials related to the natural environment. These models are more flexible than standard aggregate SFC models, for they allow modelling a variety of sectors. The model developed by Berg et al. (2015) considers an economy made up of five sectors: the government sector, the banking system, the household sectors, and two industrial sectors that produce energy and goods. The main findings of the paper can be summarised as follows: (i) a no growing economy can be associated with positive interest rates; (ii) an increase in energy prices can negatively affect the economic system by lowering real wages and aggregate demand, thus triggering a recession. Overall, the model shows hot to integrate heat emissions due to economic activities and climate change modelling.

Dafermos et al. (2017) develop a stock-flow-fund ecological macroeconomic model calibrated on global data, which combines a standard SFC framework with the flow-fund approach developed by Georgescu-Roegen (1979). In the model, the output is demand-led and finance is non-neutral. This allows considering the channels through which the monetary system, the real economy and the ecosystem, interact and affect each other. The two laws of thermodynamics are explicitly modelled. Supply constraints are determined by the exhaustion of natural resources as well as by environmental damages. Furthermore, climate change is included in the analysis and affects aggregate demand through the influence of catastrophes, global warming, and health issues, on the desired level of investment, savings, consumption and potential output. The paper focuses on two types of green finance policy: (i) a reduction in interest rates and the relaxing of credit rationing criteria on green loans, coupled with unchanged conditions on the remaining types of loans; (ii) a reduction in interest rates and the relaxing of credit rationing criteria on green loans, coupled with tighter conditions on conventional types of loans. The second policy generates better environmental results than the first policy, because of the lower economic growth rate. Particularly, a lower output level combined with a larger share of green investment create give rise to lower CO_2 emissions and therefore a lower atmospheric temperature. Finally, the leverage ratio of firms is lower under the second green finance policy, despite the lower economic growth rate. These results are due to the fact that damages derived by global warming are lower when the share of green loans increases.

Dafermos et al. (2018) aim at assessing and investigating the existing links between climate change and financial (in) stability. Using a stock-flow-fund macro model, the authors argue that an increase in the average temperature can be detrimental for firms' profitability and financial stability, possibly leading to a higher default rate and increasing the risk of systemic bank losses. The authors focus on the physical risks implied by climate change. They maintain that "climate-induced financial instability reinforces the adverse effects of climate change on economic activity" (Dafermos et al., 2018, p. 220). In addition, they consider the impact of global warming on households' portfolio choices. The latter tend to be diverted towards 'safer' and more liquid assets (because of the impact on economic agents' confidence), such as deposits and government bonds, causing in this way a decrease in corporate bonds' prices. To tackle the financial instability triggered by climate change, a green quantitative easing program, regarded as a long-term industrial policy, is proposed and discussed. The authors analyze a hypothetical scenario where central banks decide to buy a quarter of total green bonds worldwide. The policy's effectiveness is shown to vary according to the parameters of the model. More precisely, a crucial role is played by the sensitivity of investment in green capital assets to the differential between green bonds' and conventional bonds' yields. However, green QE policies usually help counter financial instability. Investment financing turns out to be less dependent on bank credit, and hence less subject to credit crunch risks. Moreover, slower climate change implies a reduced degree of economic damages. Therefore, firms' profitability is restored, liquidity problems are dampened, and the default ratio decreases.

The model developed by Deleidi et al. (2018) is based on four different theoretical approaches: (i) the Sraffian supermultiplier model; (ii) the Neo-Schumpeterian framework which emphasises the entrepreneurial role of the State; (iii) the SFC approach to macro-economic modelling; (iv) and recent developments in ecological economics literature aiming at extending post-Keynesian theories and models to deal with environmental issues. The paper aims at developing a simple analytical tool that can help examine: (i) the impact of innovation on economic growth and the ecosystem; and (ii) the impact of ecological feedbacks on economic growth and government spending effectiveness. The authors find that, in principle, government can be successful in supporting innovation and growth while slowing down natural reserves' depletion rates and tackling climate change. This requires targeting green innovations policies characterized by the highest ecological efficiency gains. More precisely, the State can actively promote green innovation, thus driving a change in the overall economic structure. However, ecological feedbacks affect government policy effectiveness. In addition, it is argued that the policy-makers are likely to be facing a conundrum in the next decade: green innovation allows for lower matter-, energy- and CO_2-intensity coefficients, but the higher investment and production levels may well frustrate these efficiency gains.

Bovari et al. (2018) combine a SFC approach with a dynamic predator-prey of the Lotka-Volterra model.[14] They analyze the challenges posed by climate change in conjunction with private indebtedness. The starting point of the analysis is as follows: climate-change mitigation is an expensive process and, given the multiple constraints imposed on public finances, the private sector is expected to carry out most of the burden. However, this can lead to a further explosion of private debt and trigger financial instability. The latter is co-caused by global warming and private indebtedness. The proposed policy approach consists of pricing carbon emissions through a carbon tax, which should incentivize firms to devote part of their production to the abatement of emissions. The authors conclude that, in spite of the +2° C target being plausibly already out of reach, an adequate carbon tax can be conducive to a reduction in carbon emissions and to the achievement of the +2.5°C objective. This result can be obtained without affecting economic growth, as long as adequate policies aiming at increasing the wage share and fostering the employment rate are also set in motion.

Finally, Monasterolo and Raberto (2018) propose a mix of fiscal and monetary policies (green sovereign bonds) that aim at tackling climate change. The analytical tool used to conduct the analysis is the so-called EIRIN model. The latter is a SFC model with neo-Schumpeterian insights, where the supply side is defined through a Leontief production function. In addition, the economy is made up of "heterogeneous economic sectors and subsectors characterized by adaptive behaviours and expectations (households, firms), heterogeneous capital goods characterized by different resource intensity, a credit sector characterized by endogenous money creation, and a foreign sector" (Monasterolo and Raberto, op. cit., p. 229). The simulations show that green sovereign bonds can significantly contribute to green investment and help reducing the import of raw materials. However, the implementation of this monetary policy can imply a short-run trade-off between positive effects in terms of green transition and the risk of wealth concentration. Focusing on green fiscal policies, incentives and taxes, climate change mitigation can come at the cost of negative feedbacks on the economy (for instance, in terms of an increase in the unemployment rate).

Notice that, unlike open-economy topics, ecological aspects were not initially covered by Godley and Lavoie (2007a) and the early SFC community. For this reason, they represent one of the most significant internal developments in SFC literature. The standard way to account for the impact on the ecosystem and ecological feedbacks is to couple the TFM and the BS with two additional matrices: the physical flow matrix, displayed by Table 3a, and the physical stock-flow matrix, shown by Table 3b. The physical flow matrix can be regarded as an "extension of the matrix that Georgescu-Roegen used in his flow of fund model" (Dafermos et al. 2017, p. 192; see also Georgescu-Roegen 1971, 1979, 1984). It is meant to capture the First and Second Laws of Thermodynamics. The former entails that matter and energy cannot be crated out of, or vanish into, thin air. The latter entails that production transforms low-entropy energy into high-entropy dissipated energy. For instance, fossil fuels are turned into thermal energy.

The physical stock-flow matrix displayed by Table 3b accounts for the changes that take place in physical stocks of material reserves, renewable and non-renewable energy reserves, atmospheric CO_2 concentration, the socio-economic stock (meaning the stock of capital goods

[14] See for example Goodwin (1967).

and housing), and other things which affect human life and well-being. The first row displays the stocks available at the beginning of each period, while the last row shows the same stocks at the end of the current period, that is, after additions and deductions have been considered. These additional tables highlight the three-fold role played by the Laws of Thermodynamics in Eco-SCF models.

> First, the First Law of Thermodynamics allows us to incorporate explicitly the harmful by-products of energy and matter transformation (CO2 emissions and hazardous material waste). […] these by-products cause the degradation of ecosystem services with feedback effects on the economy. Second, the Second Law of Thermodynamics implies that in the very long run the economic processes cannot rely on the energy produced from fossil fuels. Since the fossil fuel resources are finite and the economic processes transform the low-entropy energy embodied in these resources into high-entropy energy, sustainability requires the reliance of economic processes on renewable energy sources (even if there was no climate change). Third, by combining the laws of thermodynamics with Georgesu-Roegen's analysis of material degradation, it turns out that recycling might not be sufficient to ensure the availability of the material resources that are necessary for the economic processes. Hence, the depletion of matter needs to be checked separately. (Dafermos et al. 2017, p. 193)

In the next section, we extend a basic Eco-SFC model to the open economy. More precisely, we consider a two-country economy under a fixed exchange rate regime.

6. An Eco-2C-SFC model prototype

Eco-SFC models usually focus on a single-area economy. However, local impacts of climate change (and natural resources depletion) are likely to be uneven across countries. Besides, ecological shocks hitting one country or area can bring about indirect effects for other countries or areas, because of the interconnections of the balance of payments. To shed light on this yet-unexplored aspect, we developed a simplified ecological two-country SFC model (Eco-2C-SFC hereafter). The model is made up of 109 endogenous variables *plus* 66 exogenous variables and parameters. The full set of identities, equilibrium conditions and behavioural equations is displayed in Appendix. There are three main blocks of equations. The first block is about the open economy. It defines national income, import, export, consumption, tax payments, disposable income, wealth, financial assets (liabilities), the exchange rate, and interest rates, in each country or area. The second block defines their balance of payment components and government budgets. The third block of equations are about the ecosystem. It is made up of five sub-blocks. The first sub-block defines the evolution of matter resources and reserves over time. It also determines the socio-economic stock of each area. The second sub-block deals with (both renewable and non-renewable) energy resources and reserves. Non-renewable energy consumption is used by the third sub-block to determine CO_2 atmospheric concentration and the (predicted) average change in atmospheric temperature. The improvement of ecological efficiency (i.e. matter-, energy- and CO_2-intensity coefficients) due to technical progress is accounted for in the fourth sub-block. The last sub-block defines matter and energy depletion ratios. Besides, it calculates the proportion of gross damages due to climate change (in line with Dafermos 2017, 2018) and endogenises each propensities to import. This allows the model to consider the impact of both global warming and government (green) spending on household consumption plans.

Eco-2C-SFC key features can be summarised as follows:

a) The world economy is subdivided in two main areas, named Ecoland and Carbonland respectively.

b) Initial or baseline values of narrowly-defined economic variables and parameters (e.g. GDP, wealth stocks, propensities to consume, return rates, etc.) are identical across the two areas.
c) Natural resources endowments (matter and energy resources) are also identical across areas. For the sake of simplicity, we assume that firms located in each area can only access their domestic resources.
d) Both government budgets and balances of payments (current accounts) are perfectly balanced in the baseline scenario.
e) No production function is used to determine potential output. Neither labour force nor natural resources availability constrain output in the period considered. The economy is demand-led both in the short- and long-run. This two-fold hypothesis allows us to focus on the effects of global warming.
f) Unit prices are fixed and all economic variables are expressed in real terms (namely, at constant prices).
g) Techniques of production are different across areas, as Ecoland has lower energy- and matter-intensity coefficients (on average) compared to Carbonland.[15]
h) Similarly, Ecoland's share of renewable energy to total energy is higher, and the sensitivity of atmospheric temperature to CO_2 emissions is lower, compared to that of Carbonland.
i) Investment in fixed capital is assumed away. Hence total output is only made up of consumption, government expenditure and net export.
j) Equity, corporate bonds, bank deposits and private foreign investments are neglected. As a result, households hold their savings in form of (domestic) Treasury bills and/or cash.
k) Each central bank (or group of central banks) owns a stock of gold reserves (or US dollars), which are used to settle international payments.
l) One unit of Ecoland's currency, call it Green Dollar, is conventionally set equal to one unit of USD.
m) Carbonland's currency, the Brown Dollar, is pegged to the Green Dollar. In other words, the exchange rate is fixed.[16]

Narrowly defined economic variables are calibrated in such a way to obtain a gross world output equal to 80 trillion USD ca in the baseline scenario.[17] As a result, the baseline output of a single block, say Ecoland, roughly amounts to the combined GDP of the two biggest economic areas worldwide, namely, the US and the EU. Correspondingly, Carbonland's output amounts to the rest of the world's GDP. Economic parameters are usually taken from chapter 6 of Godley and Lavoie (2007a), while the ecological part of the model is calibrated based on Dafermos et al. (2017) and ICCP (2018).[18] The model has been run from 1960 to 2100, on an annual basis. Baseline values have been obtained based on the assumption of zero growth at worldwide level. A slow decline in matter- and energy-intensity coefficients (0.5% and 1%, respectively) has been assumed in line with available data. As a result, the change in the average atmospheric temperature relative to the 1950s is expected to be 1.5C ca in 2030, heading to 2.9C ca in 2100. A sharp decline in CO_2-intensity coefficients after 2020

[15] For the sake of clarity, when simulating the model, we assume that CO_2-intensity coefficients do not vary across areas.
[16] Assumptions (e), (f), (i), (j) and (m) have been relaxed in a more advanced version of our model.
[17] Table 4 shows coefficients and initial values of stocks at http://models.sfc-models.net/. We are happy to provide the program file of our model upon request.
[18] See the fifth column of Table 4 for information about the source of data.

(at 3% rate, in line with Paris Climate Pact of 2015) has been also considered to set baseline values. This would keep climate change below 2.5C, under a zero-growth scenario.

Model Eco-2C-SFC can now be used to check the behaviour of main endogenous variables under alternative scenarios. More precisely, we test here the effects of two changes or shocks in 2020. First, we simulate the impact generated by the decision of Ecoland's households to reduce their consumption of goods and services made in Carbonland (first scenario). This can be the effect of a higher ecological awareness of Ecoland's consumers (who turn to low-impact products, consume *zero kilometre* food, etc.). It can also result from hoarding behaviours associated with the increase in natural catastrophes' frequency, which increases uncertainty about the future. Second, we test the effects triggered by Carbonland's government decision to cut green spending – for instance, to repeal green incentives to buy photovoltaic solar panels or other green products 'made in Ecoland' (second scenario). The reason is that green incentives end up increasing imports, thus affecting Carbonland's current account balance.[19] Hence, the decision of Carbonland's government to cancel the incentives plan. These two scenarios can be regarded as two phases of the same sequence of events: climate change affects Ecoland's import of non-green products, which affects Carbonland's trade balance, which, in turn, leads Carbonland's policy-makers to reduce green spending, thus rebalancing both net export and government budget. For the sake of clarity, we analyse each scenario separately.

Figure 1 displays domestic GDPs, balances of payments, climate change, CO_2 emissions, depletion ratios, and reserves of matter and energy under the first scenario or experiment, that is, when global warming affects Ecoland consumers' propensity to import from Carbonland. As expected, the fall in Ecoland's propensity to import is associated with a fall in Carbonland's GDP – Figure 1a. The latter is exactly matched by an increase in Ecoland's GDP, which leaves the world output (quantitatively) unchanged. Carbonland now records a budget deficit, which mirrors its current account deficit – Figure 1b. This is no surprise. Since the private sector tends to balance the budget in the medium run, a persistent deficit in in the current account entails the government sector issuing more bonds. This happens because Carbonland's GDP declines, and so do tax revenues. Hence, a budget deficit shows up (unless the government revises its spending plans downwards and/or increases tax rates, but this depresses further the economy!). In a specular manner, Ecoland records a twin surplus. Figure 1c shows that the temperature reduces relative to its baseline value, due to the higher ecological efficiency of Ecolands' techniques of production relative to Carbonland's. Two aspects are worth being stressed here. First, the drop in Carbonland's CO_2 emissions outstrips the increase in Ecoland's emissions – see Figure 1d. Second, the higher share of renewable energy sources allows Ecoland's to reduce worldwide energy depletion relative to the baseline – Figure 1e.[20] Figure 1f shows that higher matter and energy reserves are available as well, thus postponing the *redde rationem* with natural resources' scarcity. To sum up, if global warming triggers a radical change in consumption habits favouring green products, this brings about a beneficial effect on the ecosystem, while leaving unchanged worldwide output and wealth. However, non-green economies are negatively affected, because of the fall in export. Paradoxically, their

[19] In our model, this effect is considered by assuming that Brownland's propensity to import is positively associated with changes in government green spending. Ecoland's propensity to import, in contrast, is a decreasing function of government green spending, as most green products are made in Ecoland. See equations (108) and (109) in Appendix (3.5) and Table 4.

[20] In principle, above effects can be further strengthened by a lower CO_2-intensity coefficient of Ecoland compared with Brownland. As mentioned, we assume away this additional effect in our simulations.

governments can be forced to adopt austerity measures (to cope with their twin deficit) exactly when more spending would be necessary to foster the transition to green technologies!

Turning to the second scenario, our findings are displayed by Figure 2. The decision of Carbonland's government to withdraw green incentives affects heavily Ecoland and the world economy. Despite the austerity, Carbonland's GDP is not necessarily affected. In fact, the new equilibrium level for the GDP can be even higher *if* the fall in the propensity to import outstrips the reduction in domestic demand. This is the case portrayed by Figure 2a. However, the overall effect on the world economy is *always* negative. Now Carbonland records a twin surplus (Figure 2b), while Ecoland faces a twin deficit. Once again, the deficit is due to the fall in tax revenues (given government spending), due to the contraction of Ecoland's GDP. The atmospheric temperature reduces, due to the sharp fall in world output – Figure 2c. More generally, the ecosystem benefits from Carbonland's government decision – see figures 2d, 2e and 2f. However, this is not due to a higher ecological efficiency of production processes. On the contrary, it is due to the collapse of international trade and the world economy. In a sense, this can be regarded as the 'low road' to ecological sustainability, as opposed to the 'high road' described by the first experiment. They both entail international imbalances that can only be addressed by means of a coordinated macroeconomic plan negotiated and adopted by the two areas.

7. Concluding remarks

The stock-flow consistent approach to macroeconomic dynamic modelling was developed in the 2000s by Godley and Lavoie (2007a, 2007b), who paved the way for the flourishing of SFC models. These models are based on sound accounting principles, which *inter alia* allow constraining models' dynamics without anchoring them to any preordained long-run supply-side equilibrium. In addition, SFC models enable accounting for the process of money creation (and destruction), while including a variety of financial assets, motives and agents. In previous sections, we provided a survey of SFC models' literature and we discussed recent developments. More precisely, we identified two types of 'external' development, or cross-fertilisations, and three types of 'internal' development of SFC models. The former aim at crossbreeding benchmark SFC models with other analytical tools and/or modelling techniques. Successful cross-fertilisations include agent-based SFC models and input-output SFC models, providing the basic macroeconomic structure with micro- and meso-foundations, respectively. By contrast, the three main types of internal developments are empirical SFC models, open-economy or multi-country SFC model, and ecological SFC models. Building upon this taxonomy, we identified a gap in current literature on SFC models, as ecological models usually focus on a single country or the world economy. For this reason, we presented an ecological 2-area SFC model prototype. Despite its simplified structure, the model enables testing a variety of shocks and comparing different scenarios. For instance, we showed that the uneven technical progress, coupled with rising ecological awareness of the 'consumers', can force governments of less ecologically-efficient areas to adopt austerity measures and/or implement protectionist policies, thus moving further away from green technologies. While the environment can benefit from these changes (in terms of a lower average temperature compared to the baseline value), this happens because both the volume of international trade and world output collapse. Arguably, these issues can be addressed by means of an internationally-coordinated macroeconomic plan, aimed at supporting green transition of 'brown' economies. Notice that the advantages of the methodology we have just presented go well beyond the specific findings of our experiment. SFC models are effective tools to assess the impact of alternative policy options in a multi-country environment, where economic and financial variables interact with the broader ecosystem.

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Tables and figures

Table 1. Balance-sheet of a simplified two-country economy

	ECOLAND (g)			CARBONLAND (c)			
	Households	**Government**	**Central bank**	**Households**	**Government**	**Central bank**	**Σ**
Money	$+H_{gh}$		$-H_{gh}$	$+H_{ch}$		$-H_{ch}$	0
Bills	$+B_{gh}$	$-B_g$	$+B_{gcb}$	$+B_{ch}$	$-B_c$	$+B_{ccb}$	0
Gold reserves			$+OR_g \cdot p_{org} \cdot E$			$+OR_c \cdot p_{orc}$	$OR_g \cdot p_{org} \cdot E + OR_c \cdot p_{orc}$
Balance (net worth)	$-V_{gh}$	$+V_{gG}$		$+V_{ch}$	$+V_{cG}$		$-(OR_g \cdot p_{org} \cdot E + OR_c \cdot p_{orc})$
Σ	0	0	0	0	0	0	0

Notes: E is the exchange rate. A '+' before a magnitude denotes an asset, whereas '–' denotes a liability (except for Balance's entries, where signs are reversed). Fixed exchange rates are assumed. No private transactions in foreign assets are allowed. Central banks held all their foreign exchange reserves in the form of gold.

Table 2. Transactions-flow matrix of a simplified two-country economy

	ECOLAND (g)					CARBONLAND (c)				
	Households	**Firms**	**Government**	**Central bank**		**Households**	**Firms**	**Government**	**Central bank**	**Σ**
Consumption	$-C_g$	$+C_g$				$-C_c$	$+C_c$			0
Gov. spending		$+G_g$	$-G_g$				$+G_c$	$-G_c$		0
Ecoland export to Carbonland		$+X_g$			$\cdot E$		$-IM_c$			0
Carbonland export to Ecoland		$-IM_g$			$\cdot E$		$+X_c$			0
GDP	$+Y_g$	$-Y_g$				$+Y_c$	$-Y_c$			0
Interests	$+r_{g,-1} \cdot B_{gh,-1}$		$-r_{g,-1} \cdot B_{g,-1}$	$+r_{g,-1} \cdot B_{gcb,-1}$		$+r_{c,-1} \cdot B_{ch,-1}$		$-r_{c,-1} \cdot B_{c,-1}$	$+r_{c,-1} \cdot B_{ccb,-1}$	0
CB profits			$+r_{g,-1} \cdot B_{g,-1}$	$-r_{g,-1} \cdot B_{gcb,-1}$				$+r_{c,-1} \cdot B_{c,-1}$	$-r_{c,-1} \cdot B_{ccb,-1}$	0

Taxes	$-T_g$		$+T_g$			$-T_c$		$+T_c$		0
Change in cash	$-\Delta H_{gh}$			$+\Delta H_{gh}$		$-\Delta H_{ch}$			$+\Delta H_{ch}$	0
Change in bills	$-\Delta B_{gh}$		$+\Delta B_g$	$-\Delta B_{gcb}$		$-\Delta B_{ch}$		$+\Delta B_c$	$-\Delta B_{ccb}$	0
Change in gold				$-\Delta OR_g \cdot p_{org}$	$\cdot E$				$-\Delta OR_c \cdot p_{orc}$	0
Σ	0	0	0	0		0	0	0	0	0

Notes: E is the exchange rate. A '+' before a magnitude denotes a receipt or a source of funds, whereas '–' denotes a payment or a use of funds. Fixed exchange rates are assumed. No private transactions in foreign assets are allowed. Central banks held all their foreign exchange reserves in the form of gold.

Table 3. Physical matrices of the economy

(a) Physical flow matrix

	Material balance	**Energy balance**
Inputs		
Extracted matter	$+mat$	
Renewable energy		$+er$
Non-renewable energy	$+cen$	$+en$
Oxygen	$+O2$	
Outputs		
Industrial CO_2 emissions	$-emis$	
Waste	$-wa$	
Dissipated energy		$-ed$

(b) Physical stock-flow matrix

	Material reserves	**Non-Renewable Energy reserves**	**Atmospheric CO_2 concentration**	**Socio-economic stock**
Initial stock	$k_{m,-1}$	$k_{en,-1}$	$co2_{AT,-1}$	$k_{se,-1}$
Resources converted into reserves	$+conv_m$	$+conv_e$		
Emissions			$+emis$	
Production of material goods				$+y_{mat}$
Extraction/use of matter/energy	$-mat$	$-en$		
Net transfer to oceans/biosphere			$+(\phi_{11}-1)\cdot co2_{AT,-1} + \phi_{21}\cdot co2_{UP,-1}$	
Destruction of socio-economic stock				$-des$

Change in socio-economic stock	$-\Delta k_{se}$						
Σ	0	0	**Final stock**	k_m	k_e	$co2_{AT}$	k_{se}

Notes: Matter is measured in Gt while energy is measure in EJ. A '+' sign denotes inputs in the socio-economic system (a) or additions to the opening stock (b), whereas '–' denotes outputs (a) or reductions (b). Hazardous waste not included.

Figure 1. Simulations: first experiment

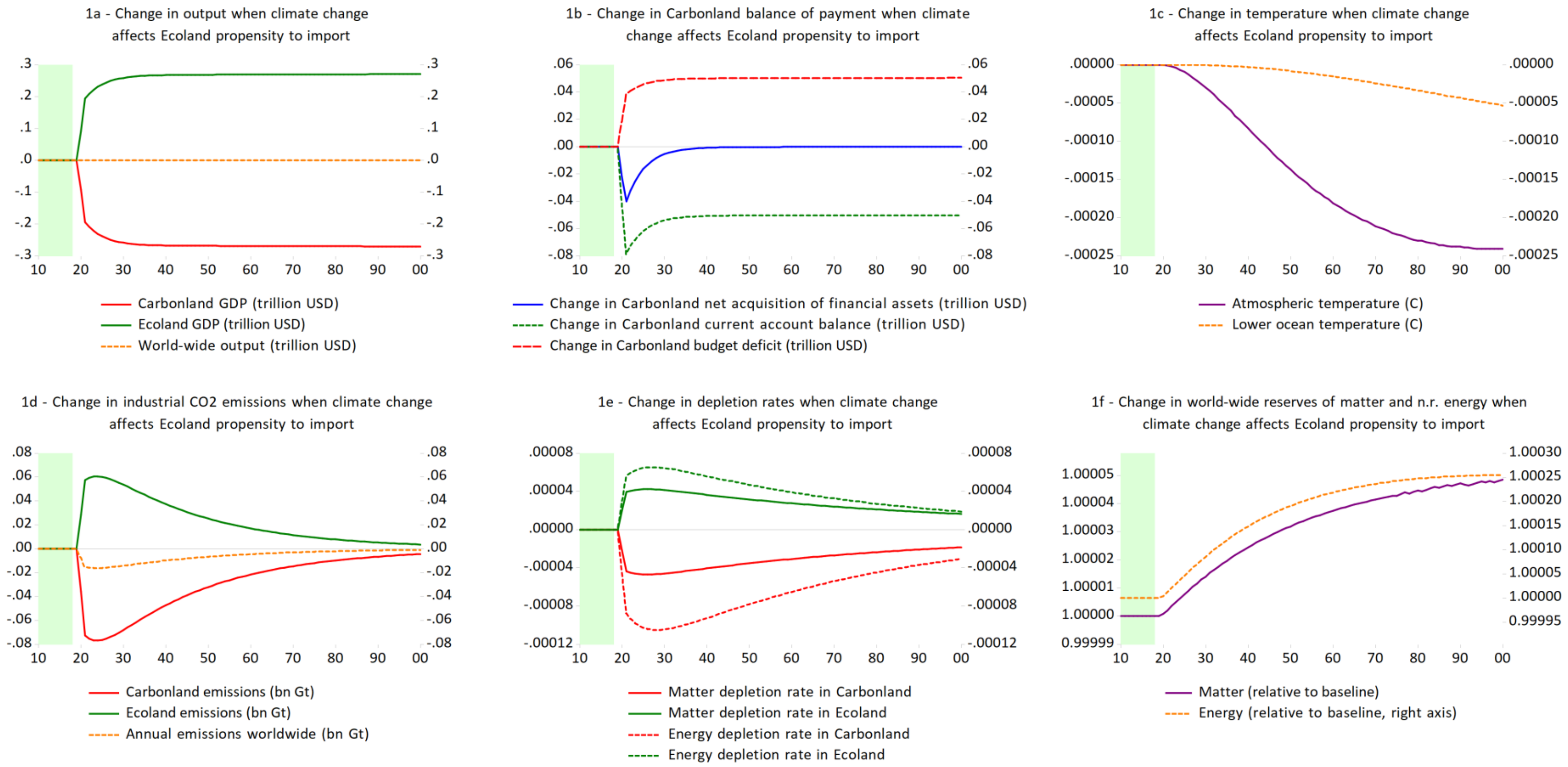
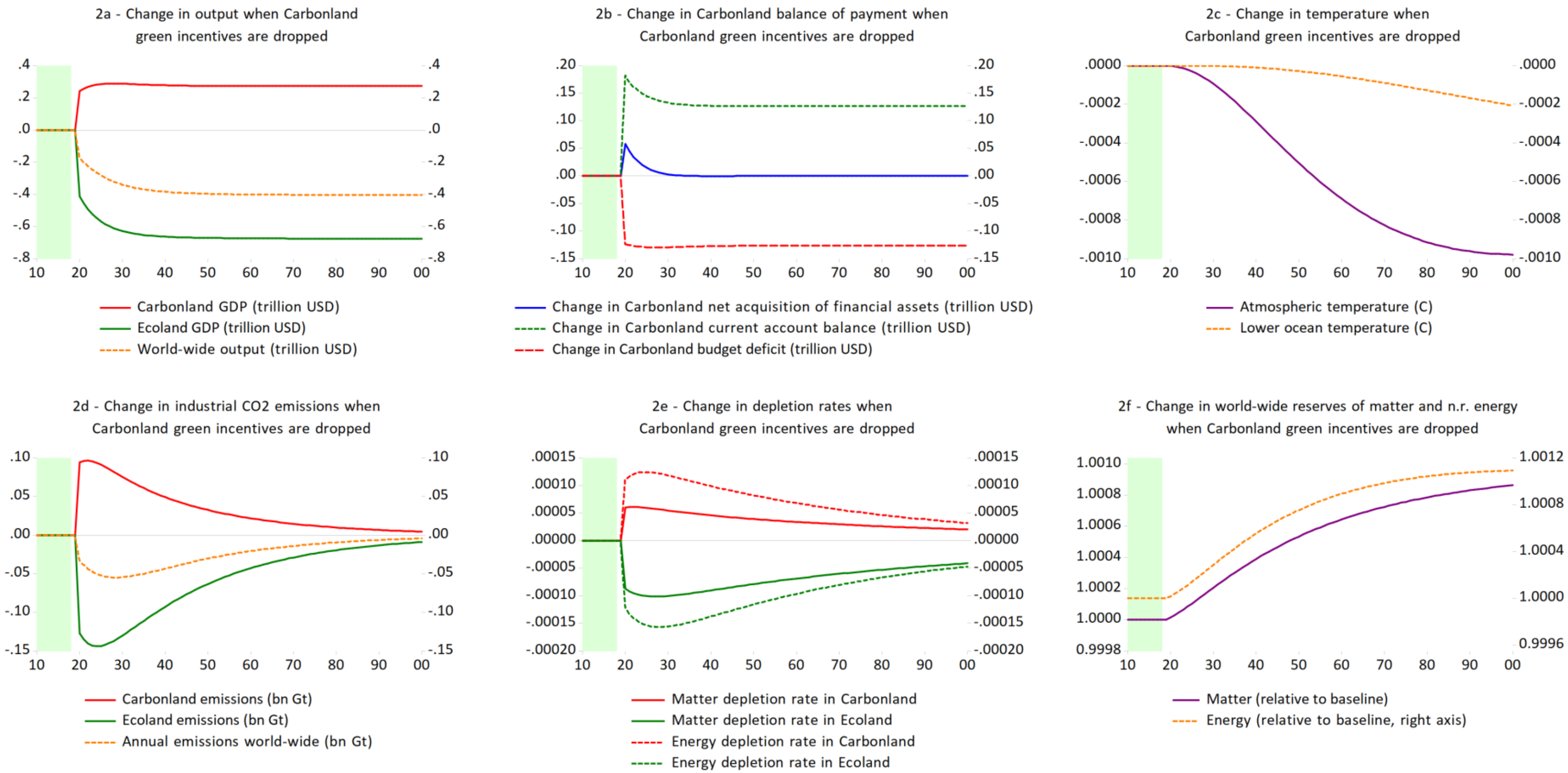


Figure 2. Simulations: second experiment



Appendix: Eco-2C-SFC model equations

The model is made up of 109 equations. Exogenous variables and parameters are 66. The model is split in four blocks of equations: basic equations of the open-economy model; equations for government budgets and balances of payment of the two areas; and equations for the ecosystem (including matter reserves, energy reserves, CO_2 emissions and climate change, ecological efficiency, depletion ratios and damages). The latter are based on Dafermos et al. (2017, 2018). The redundant equation of the system is the amount of gold bars (or USD reserves) exchanged by the central banks. All coefficient values and initial values of stocks are shown by Table 4 at http://models.sfc-models.net/.

1. Basic equations of the open-economy model

$Y_c = C_c + G_c + X_c - IM_c$	(1) National income of Carbonland
$Y_g = C_g + G_g + X_g - IM_g$	(2) National income of Ecoland
$IM_c = m_c \cdot Y_c \cdot \left(1 - \delta^c_{T,-1}\right)$	(3) Import of Carbonland
$IM_g = m_g \cdot Y_g \cdot \left(1 - \delta^g_{T,-1}\right)$	(4) Import of Ecoland
$X_c = IM_g / E$	(5) Export of Carbonland
$X_g = IM_c \cdot E$	(6) Export of Ecoland
$YD_c = Y_c - T_c + r_{c,-1} \cdot B_{hc,-1}$	(7) Disposable income in Carbonland
$YD_g = Y_g - T_g + r_{g,-1} \cdot B_{hg,-1}$	(8) Disposable income in Ecoland
$T_c = \theta_c \cdot (Y_c + r_{c,-1} \cdot B_{hc,-1})$	(9) Tax payments in Carbonland
$T_g = \theta_g \cdot (Y_g + r_{g,-1} \cdot B_{hg,-1})$	(10) Tax payments in Ecoland
$V_c = V_{c,-1} + YD_c - C_c$	(11) Wealth accumulation in Carbonland
$V_g = V_{g,-1} + YD_g - C_g$	(12) Wealth accumulation in Ecoland
$C_c = (\alpha_{c1} \cdot YD_c + \alpha_{c2} \cdot V_{c,-1}) \cdot \left(1 - \delta^c_{T,-1}\right)$	(13) Domestic consumption in Carbonland
$C_g = (\alpha_{g1} \cdot YD_g + \alpha_{g2} \cdot V_{g,-1}) \cdot \left(1 - \delta^g_{T,-1}\right)$	(14) Domestic consumption in Ecoland
$H_{hc} = V_c - B_{hc}$	(15) Cash money held in Carbonland
$H_{hg} = V_g - B_{hg}$	(16) Cash money held in Ecoland
$B_{hc} = V_c \cdot \lambda_{c0} + V_c \cdot \lambda_{c1} \cdot r_c - \lambda_{c2} \cdot YD_c$	(17) Demand for government bills in Carbonland
$B_{hg} = V_g \cdot \lambda_{g0} + V_g \cdot \lambda_{g1} \cdot r_g - \lambda_{g2} \cdot YD_g$	(18) Demand for government bills in Ecoland
$B_{sc} = B_{gc,-1} + \left(G_c + r_{c,-1} \cdot B_{gc,-1}\right) - (T_c + r_{c,-1} \cdot B^{cb}_{c,-1})$	(19) Supply of government bills in Carbonland
$B_{sg} = B_{gg,-1} + \left(G_g + r_{g,-1} \cdot B_{gg,-1}\right) - (T_g + r_{g,-1} \cdot B^{cb}_{g,-1})$	(20) Supply of government bills in Ecoland

$B_c^{cb} = B_{gc} - B_{hc}$ (21) Bills held by central bank in Carbonland

$B_g^{cb} = B_{gg} - B_{hg}$ (22) Bills held by central bank in Ecoland

$OR_c = OR_{c,-1} + \left(H_{gc} - H_{gc,-1} - \left(B_c^{cb} - B_{c,-1}^{cb}\right)\right)/p_{or,c}$ (23) Gold held by central bank in Carbonland

$OR_g = OR_{b,-1} + \left(H_{gg} - H_{gg,-1} - \left(B_g^{cb} - B_{g,-1}^{cb}\right)\right)/p_{or,g}$ (24) Gold held by central bank in Ecoland

$H_{gc} = H_{hc}$ (25) Supply of cash money in Carbonland

$H_{gg} = H_{hg}$ (26) Supply of cash money in Ecoland

$p_{or,c} = \bar{p}_{or}$ (27) Unit price of gold in Carbonland

$p_{or,g} = p_{or,b} \cdot E$ (28) Unit price of gold in Ecoland

$E = \bar{E}$ (29) Exchange rate (fixed)

$r_c = \bar{r}_c$ (30) Interest rate in Carbonland

$r_g = \bar{r}_g$ (31) Interest rate in Ecoland

Notes: m_c and m_g are the propensities to import of Carbonland and Ecoland, respectively; E is the nominal exchange rate; θ_c and θ_g are the average tax rates; α_{c1} and α_{g1} are the propensities to consume out of income; α_{c2} and α_{g2} are the propensities to consume out of wealth; λ_{c0}, λ_{c1}, λ_{c2}, λ_{g0}, λ_{g1}, and λ_{g2} are parameters of household portfolio equations.

2. Additional equations for government budgets and balances of payment

$B_s = B_{sc} + B_{sg}$ (32) Worldwide supply of government bills

$DEF_c = G_c + r_{c,-1} \cdot B_{sc,-1} - T_c - r_{c,-1} \cdot B_{c,-1}^{cb}$ (33) Government deficit of Carbonland

$DEF_g = G_g + r_{g,-1} \cdot B_{sg,-1} - T_g - r_{g,-1} \cdot B_{g,-1}^{cb}$ (34) Government deficit of Ecoland

$NAFA_c = DEF_c + CAB_c$ (35) Net accumulation of financial assets in Carbonland

$NAFA_g = DEF_g + CAB_g$ (36) Net accumulation of financial assets in Ecoland

$CAB_c = TB_c$ (37) Current account balance in Carbonland

$CAB_g = TB_g$ (38) Current account balance in Ecoland

$KABP_c = d(OR_c) \cdot p_{orc}$ (39) Financial account balance in Carbonland

$KABP_g = d\left(OR_g\right) \cdot p_{org}$ (40) Financial account balance in Ecoland

$TB_c = X_c - IM_c$ (41) Trade balance of Carbonland

$TB_g = X_g - IM_g$ (42) Trade balance of Ecoland

$BP_c = CAB_c$ (43) Balance of payments of Carbonland

$BP_g = CAB_g$ (44) Balance of payments of Ecoland

3. Equations for the ecosystem

3.1 Material resources and reserves

$y_{matc} = \mu_c \cdot Y_c$ (45) Production of material goods in Carbonland

$y_{matg} = \mu_g \cdot Y_g$ (46) Production of material goods in Ecoland

$mat_c = y_{matc} - rec_c$ (47) Extraction of matter in Carbonland

$mat_g = y_{matg} - rec_g$ (48) Extraction of matter in Ecoland

$rec_c = \rho_c \cdot dis_c$ (49) Recycled socio-economic stock in Carbonland

$rec_g = \rho_g \cdot dis_g$ (50) Recycled socio-economic stock in Ecoland

$dis_c = \mu_c \cdot (C_c - TB_c)$ (51) Discarded socio-economic stock in Carbonland

$dis_g = \mu_g \cdot (C_g - TB_g)$ (52) Discarded socio-economic stock in Ecoland

$k_{sec} = k_{sec,-1} + y_{matc} - \mu_c \cdot TB_c - dis_c$ (53) Socio-economic stock in Carbonland

$k_{seg} = k_{seg,-1} + y_{matg} - \mu_g \cdot TB_g - dis_g$ (54) Socio-economic stock in Ecoland

$wa_c = mat_c - d(k_{sec})$ (55) Waste generated in Carbonland

$wa_g = mat_g - d(k_{seg})$ (56) Waste generated in Ecoland

$k_{mc} = k_{mc,-1} + conv_{mc} - mat_c$ (57) Stock of material reserves in Carbonland

$k_{mg} = k_{mg,-1} + conv_{mg} - mat_g$ (58) Stock of material reserves in Ecoland

$k_m = k_{mc} + k_{mg}$ (59) Worldwide stock of material reserves

$conv_{mc} = \sigma_{mc} \cdot res_{mc}$ (60) Material resources converted to reserves in Carbonland

$conv_{mg} = \sigma_{mg} \cdot res_{mg}$ (61) Material resources converted to reserves in Ecoland

$res_{mc} = res_{mc,-1} - conv_{mc}$ (62) Stock of material resources in Carbonland

$res_{mg} = res_{mg,-1} - conv_{mg}$ (63) Stock of material resources in Ecoland

$res_m = res_{mc} + res_{mg}$ (64) Worldwide stock of material resources

$cen_c = emis_c/car$ (65) Carbon mass of non-renewable energy in Carbonland

$cen_g = emis_g/car$ (66) Carbon mass of non-renewable energy in Ecoland

$o2_c = emis_c - cen_c$ (67) Mass of oxygen issued by Carbonland

$o2_g = emis_g - cen_g$ (68) Mass of oxygen issued by Ecoland

Notes: μ_c and μ_g are the matter-intensity coefficients in Carbonland and Ecoland, respectively; ρ_c and ρ_g are recycling rates; σ_{mc} and σ_{mg} are rates of conversion of material resources into reserves; car is the coefficient converting Gt of carbon into Gt of CO_2.

3.2 Energy resources and reserves

$e_c = \epsilon_c \cdot Y_c$ (69) Energy required for production in Carbonland

$e_g = \epsilon_g \cdot Y_g$ (70) Energy required for production in Ecoland

$er_c = \eta_c \cdot e_c$ (71) Renewable energy in Carbonland

$er_g = \eta_g \cdot e_g$ (72) Renewable energy in Ecoland

$en_c = e_c - er_c$ (73) Non-renewable energy in Carbonland

$en_g = e_g - er_g$ (74) Non-renewable energy in Ecoland

$ed_c = er_c + en_c$ (75) Dissipated energy in Carbonland (end of period)

$ed_g = er_g + en_g$ (76) Dissipated energy in Ecoland (end of period)

$k_{ec} = k_{ec,-1} + conv_{ec} - en_c$ (77) Stock of energy reserves in Carbonland

$k_{eg} = k_{eg,-1} + conv_{eg} - en_g$ (78) Stock of energy reserves in Ecoland

$k_e = k_{ec} + k_{eg}$ (79) Worldwide stock of energy reserves

$conv_{ec} = \sigma_{ec} \cdot res_{ec}$ (80) Energy resources converted to reserves in Carbonland

$conv_{eg} = \sigma_{eg} \cdot res_{eg}$ (81) Energy resources converted to reserves in Ecoland

$res_{ec} = res_{ec,-1} - conv_{ec}$ (82) Stock of non-renewable energy resources in Carbonland

$res_{eg} = res_{eg,-1} - conv_{eg}$ (83) Stock of non-renewable energy resources in Ecoland

$res_e = res_{ec} + res_{eg}$ (84) Worldwide stock of energy resources

Notes: ϵ_c and ϵ_g are the energy-intensity coefficients in Carbonland and Ecoland, respectively; η_c and η_g are the shares of renewable energy to total energy; σ_{ec} and σ_{eg} are the rates of conversion of non-renewable energy resources into reserves.

3.3 Emissions and climate change

$emis_c = \beta_c \cdot en_c$ (85) Industrial emissions of CO_2 in Carbonland

$emis_g = \beta_g \cdot en_g$ (86) Industrial emissions of CO_2 in Ecoland

$emis_l = emis_{l,-1} \cdot (1 - g_l)$ (87) Land emissions of CO_2

$emis = emis_c + emis_g + emis_l$ (88) Total emissions of CO_2 worldwide

$co2_{AT} = emis + \phi_{11} \cdot co2_{AT,-1} + \phi_{21} \cdot co2_{UP,-1}$ (89) Atmospheric CO_2 concentration

$co2_{UP} = \phi_{12} \cdot co2_{AT,-1} + \phi_{22} \cdot co2_{UP,-1} + \phi_{32} \cdot co2_{LO,-1}$ (90) Upper ocean/biosphere CO_2 concentration

$co2_{LO} = \phi_{23} \cdot co2_{UP,-1} + \phi_{33} \cdot co2_{LO,-1}$ (91) Lower ocean CO_2 concentration

$F = F_2 \cdot \log_2\left(\frac{co2_{AT}}{co2_{AT}^{PRE}}\right) + F_{EX}$ (92) Radiative forcing over pre-industrial levels (W/m^2)

$F_{EX} = F_{EX,-1} + fex$ (93) Radiative forcing due to non-CO_2 greenhouse gases (W/m^2)

$T_{AT} = T_{AT,-1} + \tau_1 \cdot \left[F - \frac{F_2}{s} \cdot T_{AT,-1} - \tau_2 \cdot (T_{AT,-1} - T_{LO,-1})\right]$ (94) (Change in) atmospheric temperature

$T_{LO} = T_{LO,-1} + \tau_3 \cdot (T_{AT,-1} - T_{LO,-1})$ (95) (Change in) lower ocean temperature

Notes: β_c and β_g are the CO_2-intensity coefficients of production processes in Carbonland and Ecoland, respectively; g_l is the rate of decline of land-use CO_2 emissions; ϕ_{ij} are CO_2 transfer coefficients; F_2 is the increase in radiative forcing (due to doubling of CO_2 concentraton) since pre-industrial levels; $co2_{AT}^{PRE}$ is the pre-industrial CO_2 concentration; fex is the annual increase in radiative forcing due to non-CO_2 greenhouse gas emissions; τ_1 is the speed of adjustment of atmospheric temperature; τ_2 and τ_3 are coefficients of heat loss; and s is the equilibrium climate sensitivity

3.4 Ecological efficiency

$\mu_c = \mu_{c0} \cdot \left(1 + g_{\mu c}\right)^{-t}$ (96) Matter-intensity coefficient in Carbonland

$\mu_g = \mu_{g0} \cdot \left(1 + g_{\mu g}\right)^{-t}$ (97) Matter-intensity coefficient in Ecoland

$\epsilon_c = \epsilon_{c0} \cdot (1 + g_{\epsilon c})^{-t}$ (98) Energy-intensity coefficient in Carbonland

$\epsilon_g = \epsilon_{g0} \cdot \left(1 + g_{\epsilon g}\right)^{-t}$ (99) Energy-intensity coefficient in Ecoland

$\beta_c = \beta_{c0} \cdot \left(1 + g_{\beta c}\right)^{-t}$ (100) CO_2-intensity coefficient in Carbonland

$\beta_g = \beta_{g0} \cdot \left(1 + g_{\beta g}\right)^{-t}$ (101) CO_2-intensity coefficient in Ecoland

Notes: g_{ic} and g_{ig} (with $i = \mu, \epsilon, \beta$) define the rates of reduction over time of matter-, energy- and CO_2-intensity coefficients of Carbonland and Ecoland, respectively; subscript '0' refers to initial values of variables.

3.5 Depletion ratios, damages and feedbacks

$\delta_{mc} = mat_c / k_{mc}$ (102) Matter depletion ratio in Carbonland

$\delta_{mg} = mat_g / k_{mg}$ (103) Matter depletion ratio in Ecoland

$\delta_{ec} = en_c / k_{ec}$ (104) Energy depletion ratio in Carbonland

$\delta_{eg} = en_g / k_{eg}$ (105) Energy depletion ratio in Ecoland

$\delta_T^c = 1 - \left(1 + d_1^c \cdot T_{AT} + d_2^c \cdot T_{AT}^2 + d_3^c \cdot T_{AT}^{x_c}\right)^{-1}$ (106) Proportion of gross damage in Carbonland due to changes in temperature

$\delta_T^g = 1 - \left(1 + d_1^g \cdot T_{AT} + d_2^g \cdot T_{AT}^2 + d_3^g \cdot T_{AT}^{x_g}\right)^{-1}$ (107) Proportion of gross damage in Ecoland due to changes in temperature

$m_c = m_{c,-1} + m_0^c + m_1^c \cdot (G_c - G_{c,-1})$ (108) Carbonland propensity to import

$m_g = m_{g,-1} + m_0^g - m_1^g \cdot (G_g - G_{g,-1})$ (109) Ecoland propensity to import

Notes: d_i^j and x_j (with $i = 1,2,3$ and $j = c, g$) are positive coefficients such that: $0 < \delta_T^j < 1$ and $T_{AT} = 6 \rightarrow \frac{\delta_T^c + \delta_T^g}{2} = 0.5$; m_i^j (with $i = 0,1$ and $j = c, g$) are positive coefficients.

Redundant equations

$\Delta OR_c = -\Delta OR_g$ Zero reserve gains (losses) across areas